

## Math 129-8H Written Homework #3

Due September 17, in class.

1. Some functions are defined as integrals. Consider the functions  $F_k$  defined by

$$F(x) = \int_0^x \sqrt{1 - 4 \sin^2 t} dt.$$

These functions are called elliptic functions of the second kind and can be thought of as just some other set of functions like  $e^x$ ,  $\sin x$ , etc. In fact, you can find tables of values for these functions in standard mathematical reference books. Notice that  $F(0) = 0$ . Find the following integrals in terms of the functions  $F_k$  by using integration by parts or substitution. Answers should be a number like  $3e^2 + F(2) - \frac{1}{2}F(\pi)$  or something like that.

(a)  $\int_0^2 \sqrt{1 - 4 \sin^2 t} dt$

(b)  $\int_0^2 t \sqrt{1 - 4 \sin^2(t^2)} dt$

(c)  $\int_0^1 \frac{t \sin t \cos t}{\sqrt{1 - 4 \sin^2 t}} dt$  (Hint: consider the derivative of  $\sqrt{1 - 4 \sin^2 t}$  and then do integration by parts).

2. Evaluate the following antiderivatives:

(a)  $\int \sin^4 t dt$

(b)  $\int \sin^5 t dt$

(c)  $\int \frac{1}{x^2 + 2x + 1} dx$

(d)  $\int \frac{1}{x^2 + 2x - 1} dx$

(e)  $\int \frac{1}{x^2 + 2x + 4} dx$

3. 7.3, problem number 46.