Math 129-8H Written Homework #5

Due October 1, in class.

1. 7.7, numbers 10, 12, 22, 36
2. 7.8, numbers 14, 16, 24, 28
3. We can compute

$$\int_{-\infty}^{\infty} e^{-x^2} dx$$

in the following ingenious way. We will integrate

$$\begin{aligned}\left(\int_{-\infty}^{\infty} e^{-x^2} dx\right)^2 &= \left(\int_{-\infty}^{\infty} e^{-x^2} dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2} dy\right) \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2} e^{-y^2} dxdy \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\left(x^2+y^2\right)} dxdy\end{aligned}$$

where when the inside integral is performed, we hold y constant as a parameter. This is an integral over the whole xy-plane. We can convert this to an integral over the whole plane in polar coordinates as

$$\begin{aligned}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\left(x^2+y^2\right)} dxdy &= \int_0^{2\pi}\left(\int_0^{\infty} e^{-r^2} r dr\right) d\theta \\ &= 2\pi \int_0^{\infty} e^{-r^2} r dr.\end{aligned}$$

(You'll have to take my word that $dxdy = rdrd\theta$, since it won't be covered until Calculus 3.) Thus we have derived that

$$\left(\int_{-\infty}^{\infty} e^{-x^2} dx\right)^2 = 2\pi \int_0^{\infty} e^{-r^2} r dr.$$

Finish the derivation by computing the integral on the right and then taking a square root to compute

$$\int_{-\infty}^{\infty} e^{-x^2} dx.$$