Math 129-8H Written Homework #9

Due November 5, in class.

1. The catenary is the curve $y = \cosh x = \frac{1}{2}(e^x + e^{-x})$ (NOTE: the sign was wrong on the test). It is the shape of a hanging wire with evenly distributed mass.

   (a) Using a Taylor approximation centered at zero, which parabola is closest to this catenary?

   (b) Compute the arclength of the catenary between -1 and 1 and also the arclength of the parabola determined in part a between -1 and 1. Which is longer?

2. By recognizing Taylor series, compute the following sums (answers may look like $\sin 3$, etc.):

   (a) $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$

   (b) $2 - \frac{9}{2!} + \frac{81}{4!} - \frac{3^6}{6!} + \cdots$

   (c) $\sum_{n=0}^{\infty} \frac{3}{n!}$.

3. The function $\mathrm{Si}(x)$ (which stands for sine integral) is defined by

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t} dt.$$

   Find the Taylor series for $\mathrm{Si}(x)$ centered at $x = 0$. You may assume that $\mathrm{Si}(0) = 0$.