# Practice for Exam 2

1. Indicate if the following are true or false. If false, correct them in the best possible way (i.e. give as much information as possible; for instance "University of Arizona is in Yuma" should be corrected as "University of Arizona is in Tucson," not as "University of Arizona is *not* in Yuma.")

a) If a function $f(x)$ is decreasing on $[-2,2]$, then $\int_{-2}^{2} f(t)\,dt$ is bounded below by MID(100) and bounded above by TRAP(100).

False. One way to fix is to replace decreasing by concave up. Another is to replace MID by RIGHT and TRAP by LEFT.

b) $\int_1^\infty \frac{1}{x}dx$ is divergent but $\int_0^1 \frac{1}{x}dx$ is convergent.

False. Both diverge.

c) If $0 \le f(x) \le g(x)$ and $\int_1^\infty f(x)\,dx$ converges, then $\int_1^\infty g(x)\,dx$ diverges.

False. If $\int_1^\infty f(x)\,dx$ diverges, then $\int_1^\infty g(x)\,dx$ diverges. Or another possibility is: $\int_1^\infty g(x)\,dx$ converges, then $\int_1^\infty f(x)\,dx$ converges.

d) If $h(x) \ge x^{-1/2}$ then $\int_0^2 h(x)\,dx$ converges.

False. If $h(x) \le x^{-1/2}$ then $\int_0^2 h(x)\,dx$ converges.

2. Compute the following definite integrals. If an improper integrals diverges, simply answer "diverges."

a) $\int_1^\infty \frac{4}{e^{3t}}dt = \frac{4}{3}e^{-3}$

b) $\int_0^1 x^{-1/4}\ dx = \frac{4}{3}$

c) $\int_{-\infty}^\infty e^{-y}dy$ diverges.

3. a) Compute the volume of a solid whose base is the region bounded by $y = x^2 - 4$ and the $x$-axis and whose cross-sections perpendicular to the x-axis are equilateral triangles.

Answer: $\int_{-2}^{2} \left(4 - x^2\right)^2 \frac{\sqrt{3}}{4}dx = \frac{128}{15}\sqrt{3}.$

b) Now compute the volume of the solid with the same base but whose cross-sections perpendicular to the y-axis are equilateral triangles.

Answer: $\int_{-4}^{0} 4\,(4+y)\,\frac{\sqrt{3}}{4}dy = 8\sqrt{3}$

4. Guess whether the following converge or diverge. Then show it (you must actually show the comparison, not just give an idea of why you think it is based on looking at highest powers, etc):

a) $\int_2^\infty \frac{1}{x+1+\sin x}dx \ge \int_2^\infty \frac{1}{x+2}dx$, so diverges.

b) $\int_1^\infty \frac{1}{x^{10}+2x}dx \le \int_1^\infty \frac{1}{x^{10}}dx$ so converges.

c) $\int_0^2 \frac{\sqrt{x^2+14}}{x^3}dx \ge \sqrt{14}\int_0^1 \frac{1}{x^3}dx$ so diverges.

d) $\int_1^\infty e^{-x^2}dx \le \int_1^\infty e^{-x}dx$ so converges.

5.Let $R$ be the region bounded by the curve $y = x^2$, the $x$-axis, and the lines $x = 1$ and $x = 2$. Compute the volumes of the following solids:

a) the solid defined by rotating $R$ about the $y$-axis.

Answer: $\pi \int_1^4 \left(4 - \left(\sqrt{y}\right)^2\right) dy + \pi\,(4-1)\,(1) = \frac{15}{2}\pi$

b) the solid defined by rotating $R$ about the $x$-axis.

Answer: $\pi \int_1^2 \left(x^2\right)^2 dx = \frac{31}{5}\pi$

c) the solid defined by rotating $R$ about the line $y = 6$.

Answer: $\pi \int_1^2 \left(6^2 - \left(x^2\right)^2\right) dx = \frac{149}{5}\pi$

d) the solid whose base is $R$ and whose cross-sections perpendicular to the $x$-axis are circles.

Answer: $\pi \int_1^2 \left(\frac{x^2}{2}\right)^2 dx = \frac{31}{20}\pi$

e) the solid whose base is $R$ and whose cross-sections perpendicular to the $y$-axis are equilateral triangles.

Answer: $\frac{\sqrt{3}}{4} \int_1^4 \left(2 - \sqrt{y}\right)^2 dy + \frac{\sqrt{3}}{4}\,(1) = \frac{11}{24}\sqrt{3}$

6. Chapter 7 Check Your Understanding (p. 389) 9-11, 13-20,29, 30

7. Chapter 8 Check Your Understanding (p. 459) 1-8

8. Chapter 8 Review (p. 454) 55a

9. p. 422, 14.