Practice for Test 3

1) Chapter 8 Check Your Understanding: 13, 15, 16

2) Chapter 8 Review: 57, 59, 61

3) Chapter 9 Check Your Understanding: 5, 9, 11, 12, 13, 15, 17, 19, 21, 23, 27, 29, 33, 37, 39, 41, 43, 45

4) Chapter 9 Review: 23, 25, 27, 29, 31, 35, 37, 49, 51, 57

5) Chapter 10 Check Your Understanding: 1, 3, 5, 7

6) Chapter 10 Review: 9, 11, 15, 28, 30, 33, 35, 41

7) Chapter 11 Check Your Understanding: 1, 3, 5, 8, 9, 10

8) Chapter 11 Review: 1, 2, 5, 7, 9, 11, 15, 23, 25, 33

9) State if the following are true or false. Be sure to justify your answers and correct if false.

a) If the power series $\sum_{n=0}^{\infty} b_n (x-4)^n$ has radius of convergence 5 then the series converges for $-5 \leq x \leq 5$.

False: If it has radius of convergence 5, it converges for $-1 < x < 9$ (since it is centered at 4).

b) The Taylor series for $\frac{1}{1-x}$ centered at $x = 0$ has radius of convergence equal to infinity.

False: We know it has radius of convergence equal to 1.

10) Write the first four nonzero terms in the Taylor series for the following functions.

a) $\sin(x)$ around $x = 0$. Taylor polynomial is $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$

b) $\cos 3x$ around $x = 0$. Taylor polynomial is $1 - \frac{3^2x^2}{2!} + \frac{3^4x^4}{4!} - \frac{3^6x^6}{6!}$

c) xe^x around $x = 0$. Taylor polynomial is $x + x^2 + \frac{x^3}{2} + \frac{x^4}{3!}$

d) $\frac{1}{x}$around $x = 1$. Taylor polynomial is $1 - (x-1)^1 + (x-1)^2 - (x-1)^3$.

e) $\arctan(x)$ around $x = 0$. Taylor polynomial is $x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7$

f) $\ln(-x)$ around $x = -1$. Taylor polynomial is $-(x+1) - \frac{1}{2}(x+1)^2 - \frac{1}{3}(x+1)^3 - \frac{1}{4}(x+1)^4$

g) $x^4 + 2x$ around $x = 0$.Taylor polynomial is $2x + x^4$

h) $x^4 + 2x$ around $x = 2$. Taylor polynomial is $20 + 34(x-2) + 24(x-2)^2 + 8(x-2)^3$.

f) $\frac{1}{(1-x)^3}$ around $x = 0$. Taylor polynomial is $1 + 3x + 6x^2 + 10x^3$

11) Find the following by recognizing the appropriate Taylor series.

a) $1 + (0.1)^2 + (0.1)^3 + (0.1)^4 + (0.1)^5 + \cdots = \frac{1}{1-.1} = \frac{10}{9}$

b) $\sum_{n=1}^{\infty} \frac{(0.3)^n}{n} = -\ln(0.7) = \ln\frac{10}{7}$

c) $1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} - \cdots = e^{-1}$

d) $1 + x^2 + x^4 + x^6 + x^8 + \cdots = \frac{1}{1-x^2}$

e) $\sum_{n=1}^{\infty} 2^n \frac{x^n}{n!} = e^{2x} - 1$

f) $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^{2n+3}}{(2n+3)!} = \sin x - x$

12) Decide which of the following series converge and which diverge. Compute the sum if possible. Justify your answer:

$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2}\frac{1}{1-\frac{1}{2}} = 1$. Geometric series with ratio $\frac{1}{2} < 1$.

$\sum_{n=1}^{\infty} \frac{1}{2^2}$ Diverges (limit of terms is not zero).

$\sum_{n=1}^{\infty} \frac{1}{n^{1.2}}$ converges (p-test with $p > 1$ or integral test)

$\sum_{n=1}^{\infty} n^{-1}$ diverges (harmonic series or p-test with $p \leq 1$ or integral test)

$\sum_{n=1}^{\infty} \frac{1}{n^{0.2}}$ diverges (p-test with $p \leq 1$ or integral test

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-test with $p > 1$ or integral test

$\sum_{n=1}^{\infty} \frac{1}{n \ln n}$ diverges (integral test)

$\sum_{n=1}^{\infty} \frac{1}{e^n} = \frac{1}{e}\frac{1}{1-\frac{1}{e}}$. Geometric series with ratio $\frac{1}{e} < 1$.

13) True/False, if false correct:

a) All geometric series converge. False. Geometric series with ratio r, $|r| < 1$, converge.

b) For any p, the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges. False, $p > 1$ converge.

c) The density function and the cumulative distribution function are the same. Should not be there.

d) If $\int_1^{\infty} f(x)\,dx$ converges, then $\sum_{n=1}^{\infty} f(n)$ converges. False, need $f(x)$ to be positive and decreasing, then true.

e) The sequence $1, 1/2, 1/3, 1/4, 1/5, \ldots$ converges. True, converges to 0.

f) The series $\sum_{n=2}^{\infty} \frac{n^2-1}{n^2}$ converges. Diverges since the limit of the terms is 1.

g)

$$\sum_{n=2}^{9} 7\frac{1}{3^n} = \frac{7\left(1-\frac{1}{3^9}\right)}{1-\frac{1}{3}}$$

False:

$$\sum_{n=2}^{9} 7\frac{1}{3^n} = \frac{7\left(1-\frac{1}{3^8}\right)}{1-\frac{1}{3}}$$

14) Compute the radius of convergence for the following power series.

a) $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)^2}(x-3)^n$. Radius of convergence is 1.

b) $\sum_{n=0}^{\infty} \frac{9^n (n+1)^3}{n!}(x+1)^n$. Radius of convergence is ∞.

c) $\sum_{n=3}^{\infty} n(-3)^n x^{2n+1}$. Radius of convergence is $1/\sqrt{3}$.

d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^3}(x-4)^{2n}$. Radius of convergence is 1.

15) a) Use Taylor series to put the following functions in order from smallest to largest for positive values of x near zero: $1-x+x^2$, $\frac{1}{1+x}$, e^{-x}, $\cos x$, $1-x$

$1-x \leq e^{-x} \leq 1-x+x^2 \leq \frac{1}{1+x} \leq \cos x$

b) Now do the same for negative values of x near zero.

$\cos x \leq 1-x \leq e^{-x} \leq 1-x+x^2 \leq \frac{1}{1+x}$

16) a) Write $5-12i$ in the form $R\ e^{i\theta}$. $13e^{i\left(-\arctan\frac{12}{5}\right)}$

b) Write $6e^{i(5\pi/4)}$ in the form $a+bi$. $6\cos\frac{5\pi}{4} + 6i\sin\frac{5\pi}{4} = -3\sqrt{2} - 3i\sqrt{2}$

c) Using $\left(e^{i\theta}\right)^2 = e^{2i\theta}$, show that

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta.$$

d) By differentiating $e^{i\theta}$, derive the derivatives of $\cos\theta$ and $\sin\theta$.