Differential Equations

November 13, 2013

1. Easy: Show that $y = x^2$ is a solution to $xy' = 2y$.

   Harder: Find the values of k for which $y = x^2 + k$ is a solution to $2y - xy' = 6$.

2. Easy: Show that $y = e^{kx}$, where k is a constant, is a solution to $\frac{dy}{dx} = ky$.

   Harder: Show that for any constants A, $y = Ae^{kx}$ is a solution to $\frac{dy}{dx} = ky$.

   Hard: Find a solution to $\frac{dy}{dx} = ky$ that satisfies $y(0) = 6$.

3. Easy: Show that $y = \sin \omega t$, where ω is a constant, is a solution to $\frac{d^2y}{dt^2} + \omega^2 y = 0$.

   Easy: Show that $y = \cos \omega t$, where ω is a constant, is a solution to $\frac{d^2y}{dt^2} + \omega^2 y = 0$.

   Harder: Show that for any constants A and B, $y = A \sin \omega t + B \cos \omega t$ is a solution to $\frac{d^2y}{dt^2} + \omega^2 y = 0$.

   Hard: Find a solution to $\frac{d^2y}{dt^2} + \omega^2 y = 0$ that satisfies $y(0) = 2$, $\frac{dy}{dt}(0) = 3$.

4. Harder: For which of the following differential equations is $y = 2x$ a solution?
   a. $\frac{dy}{dx} = 2$

   b. $\frac{dy}{dx} = y/x$

   c. $\frac{d^2y}{dx^2} = 0$

   d. $\frac{d^3y}{dx^3} = y - x$

   e. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 6$