

Differential Equations 3

December 3, 2013

1. A turkey is taken from the refrigerator (where the temperature is 40°F) and put directly in an oven preheated to 375°F . After one hour, the turkey shows a temperature of 90°F .

a. Newton's law of heating says that an object heats at a rate proportional to the difference between its own temperature and the temperature of its surroundings. This can be described by

$$\frac{dP}{dt} = k(P - 375)$$

where P is the temperature in degrees Fahrenheit, t is the time in hours, and k is a constant of proportionality Explain if k should be positive, negative, or zero.

b. Solve the equation above, using all the information to solve for all unknown constants.

c. The turkey is done when the temperature is 180°F . How long will it need to be cooked?

2. a. Suppose you have a bank account that gives 1% yearly interest compounded continuously. Write down a differential equation for $P(t)$, the amount of money in your account where t is in years.

b. If you do not touch the account and it starts with \$10,000, how much will it have after 3 years? After 10?

c. The annual percentage yield (APY) is the rate r such that if $P(t)$ is the amount of money in the account, $P(t) = (1 + r)^t P(0)$. Calculate the APY for the account in parts a and b.

d. Suppose you were taking out \$300 per month, derive a differential equation for $P(t)$ with the same interest rate and initial amount of money as in parts a and b. Solve the equation.

e. Suppose you wanted to liquidate the account in 10 years by taking out the same amount every month. How much should be taken out per month?