Slicing and density

October 7, 2013

1) Easy: Calculate the area of a disk of radius 50 by slicing it in concentric circles and approximating the area of the slice at radius r by the length of the circle of radius r times the thickness of the slice Δr.

Harder: Suppose we have a city, Ringsburg, with a population density given by a function $f(r)$ of the distance from the city center. Suppose that density function were $f(r) = 10(15 - r)$ people per square mile for the distance up to 15 from the city center. Calculate the population of Ringsburg.

2) A rod has length 2 meters. At a distance x meter from its left end, the density of the rod is given by

$$\delta(x) = 2 + 6x \text{ g/m}.$$

Find the mass of the rod.

3) The air density h meters above the Earth's surface is

$$f(h) = 1.28e^{-0.000124h} \text{ kg/m}^3.$$

Find the mass of a cylindrical column of air 2 meters in diameter and 25 km high with base on the surface of the Earth.

4) A storage shed is in the shape of a pyramid with square base of length 100 ft and height 300 ft.

Easy: Compute the volume of the shed.

Harder: Suppose the shed is filled with sawdust whose density (in lbs/ft^3) is proportional to the vertical distance from the top, with proportionality constant k. Calculate the total mass of sawdust in the shed.

5) A storage shed is in the shape of a half cylinder (on its side) with diameter 20 ft and length 50 ft.

Easy: Compute the volume of the shed.

Harder: Suppose the shed is filled with sawdust whose density (in lbs/ft^3) is proportional to the vertical distance from the top, with proportionality constant k. Calculate the total mass of sawdust in the shed.

In class problems:

1) The density of a triangular plate with base 2 cm and height 1 cm is $\delta(x) = 1 + x$ g/cm^2, where x is the distance from the base. Find the mass.

2) The density of oil in a circlular oil slick on the surface of the ocean at a distance r meters from the center of the slick is given by $\delta(r) - 50/(1 + r)$ kg/m^2. If the slick extends from $r = 0$ to $r = 10,000$ m, find the mass of the oil.

3) Suppose the density of cars along a particular stretch of road of distance 50 miles is $\delta(x)$ cars/mile, where x is distance along the road from the southernmost point. Calculate the number of cars on the road.

4) Suppose a city is in the shape of a circle and its density is $\delta(r)$ people per square mile is a function of the distance from the city center. If the city has radius R miles, how many people live there? What if the density were $\varepsilon(x)$ people per square mile, where x is the distance from a straight road that runs right through the center of town?