

Series convergence and divergence.

For the following, determine whether or not the series converges. Explain how you determined this (which test you used, and how you use the test).

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1.

a. Easy:

$$\sum_{n=1}^{\infty} \frac{1}{n^4}.$$

b. Easy:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}.$$

c. Harder:

$$\sum_{n=0}^{\infty} \frac{1}{n^4 + 4}$$

d. Harder:

$$\sum_{n=4}^{\infty} \frac{2^n}{n^4 + 4}$$

e. Hard:

$$\sum_{n=4}^{\infty} \frac{2^n}{(n^4 + 4)n!}$$

2.

a. Easy:

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}.$$

b. Easy:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}.$$

c. Harder:

$$\sum_{n=2}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$$

d. Harder:

$$\sum_{n=1}^{\infty} \frac{\sqrt{3n-2}}{100\sqrt{n}}$$

e. Hard:

$$\sum_{n=1}^{\infty} \frac{2 + \sin(1/n)}{\sqrt{n}}$$

3.

a. Easy:

$$\sum_{n=1}^{\infty} \frac{3^n}{4^n}.$$

b. Easy:

$$\sum_{n=1}^{\infty} \left(\frac{1}{n^2} + \frac{3^n}{4^n} \right).$$

c. Harder:

$$\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} + \frac{3^n}{4^n} \right)$$

d. Harder:

$$\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{\sqrt{n}} + \frac{3^n}{4^n} \right)$$

e. Hard:

$$\sum_{n=1}^{\infty} \frac{2 + \sin(1/n)}{\sqrt{n}}$$

4.

a. Easy:

$$\sum_{n=0}^{\infty} \frac{1}{n!}.$$

b. Easy:

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}.$$

c. Harder:

$$\sum_{n=0}^{\infty} \frac{1}{(2n)!}.$$

d. Harder:

$$\sum_{n=0}^{\infty} \frac{n^2}{(2n)!}$$

e. Hard:

$$\sum_{n=0}^{\infty} \frac{n^2 3^{3n}}{(2n)!}$$