Power series and Taylor series 1

October 31, 2013

1. Find the radius of convergence and interval of convergence for the following power series:

a. Easy:

$$\sum_{n=0}^{\infty} \frac{x^n}{n^2}.$$

b. Easy:

$$\sum_{n=0}^{\infty} \frac{(x+3)^n}{n^2}.$$

c. Harder:

$$\sum_{n=0}^{\infty} \frac{x^n}{n}$$

d. Harder:

$$\sum_{n=0}^{\infty} \frac{(x+3)^n}{n!}$$

e. Hard:

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{3^n}$$

f. Hard:

$$\sum_{n=0}^{\infty} (n!)\, x^n$$

2. Find the Taylor polynomial of degree 5 and the Taylor series for the following functions centered at the given points. Compute the radius of convergence.

a. Easy: e^x centered at 0.

b. Easy: $1/(1+x)$ centered at 0.

c. Harder: $x^3 + x^6$ centered at 2.

d. Harder: $\frac{1}{x}$ centered at 1.

e. Harder: $\frac{1}{x}$ centered at -1.

f. Hard: $\sin x$ centered at 0.

g. Hard: $\cos x$ centered at 0.

h. Hard: $\ln(1+x)$ centered at 0.

i. Hard: $\arctan(x)$ centered at 0.