

Review for Test 2

1. Chapter 7 Check Your Understanding: 18, 20, 25,27
2. Chapter 8 Check Your Understanding: 1, 2, 3, 4,13, 15
3. Chapter 9 Check Your Understanding: 14, 22, 23, 24, 25
4. Chapter 9 Review: 16
5. Chapter 8 Review: 7, 11, 13, 19, 21

6. Show that the following integrals converge or diverge (you must actually show the comparison, not just give an idea of why you think it is based on looking at highest powers, etc):

- a) $\int_2^{\infty} \frac{1}{x+1+\sin x} dx$
- b) $\int_1^{\infty} \frac{1}{x^{10}+2x} dx$
- c) $\int_0^2 \frac{1}{x^3+2} dx$
- d) $\int_1^{\infty} e^{-x^2} dx$

7. Write an integral that represents the arclength of the following curves:

- a) $y = \sin x$ between $x = 0$ and $x = 2\pi$.
- b) $y = x^3 + 1$ between $x = -1$ and $x = 1$.
- c) the parametric curve $x = e^{2t}$, $y = \cos t$ for $t \in [0, 3]$.
- d) the ellipse $x = 2 \cos t$, $y = 4 \sin t$ for $t \in [0, 2\pi]$.
- e) $y = \ln x$ between $x = 1$ and $x = 2$.

8. Let R be the region bounded by the curve $y = x^2$, the x -axis, and the lines $x = 1$ and $x = 2$. Compute the volumes of the following solids:

- a) the solid defined by rotating R about the y -axis.
- b) the solid defined by rotating R about the x -axis.
- c) the solid defined by rotating R about the line $y = 6$.
- d) the solid whose base is R and whose cross-sections perpendicular to the x -axis are circles.
- e) the solid whose base is R and whose cross-sections perpendicular to the y -axis are equilateral triangles.

9. Compute the following integrals:

- a) $\int x^2 \ln x dx$
- b) $\int 2xe^{-x^2} dx$
- c) $\int \frac{1}{x^2-1} dx$
- d) $\int \frac{1}{x^2+1} dx$
- e) $\int \frac{x^2}{1-x} dx$
- f) $\int (x^2 + 1) \cos x dx$
- g) $\int \frac{x}{\sqrt{1-2x^2}} dx$
- h) $\int \frac{1}{\sqrt{1-2x^2}} dx$

10. Suppose a chain is hanging over the side of a platform. If the chain is 3 m long and its mass is 6 kg/m, how much work is required to pull the chain up?

11. Consider a fish tank which is in the shape of a cube which is 4 ft on each side. Suppose the tank is completely full of water. How much pressure is on the bottom? How much force is exerted on each side? (Recall that the density of water is 62.4 lb/ft^3).

12. How much work does it take to pump out 28 cubic feet of water from the top of a rectangular container 15 feet high with a square base which is 2 feet by 2 feet (so the water is 7 feet high)?

13. Do the rest of the problems from the chapter 8 review sheet (you know you didn't do them all!)

14. Decide which of the following series converge and which diverge. Justify your answer:

$$\sum_{n=1}^{\infty} \frac{1}{2^n}, \sum_{n=1}^{\infty} \frac{1}{2^2}, \sum_{n=1}^{\infty} \frac{1}{n^{1.2}}, \sum_{n=1}^{\infty} n^{-1}, \sum_{n=1}^{\infty} \frac{1}{n^{0.2}}, \sum_{n=1}^{\infty} \frac{1}{n^2}, \sum_{n=1}^{\infty} \frac{1}{n \ln n}, \sum_{n=1}^{\infty} \frac{1}{e^n}$$

15. True/False, if false correct:

- All geometric series converge.
- For any p , the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges.
- The units kg/m^2 can denote pressure.
- Force and pressure are the same thing.
- Mass and weight are the same thing.
- Force and weight use the same units.
- If $\int_1^{\infty} f(x) dx$ converges, then $\sum_{n=1}^{\infty} f(n)$ converges.
- The sequence $1, 1/2, 1/3, 1/4, 1/5, \dots$ converges.
- The series $\sum_{n=2}^{\infty} \frac{n^2-1}{n^2}$ converges.