7.6

8) We know that the ratio of the errors is 2 because LEFT(20) has twice as many subdivisions as LEFT(10). Hence if I represents the actual value of the integral,

$$\begin{aligned} \frac{I - LEFT\,(10)}{I - LEFT\,(20)} &= 2 \\ I - LEFT\,(10) &= 2\,(I - LEFT\,(20)) \\ 2\ LEFT\,(20) - LEFT\,(10) &= I \end{aligned}$$

Thus the actual error for LEFT(10) is $I - LEFT\,(10) = 2\,(LEFT\,(20) - LEFT\,(10)) = 2\,(.36517 - .38745) = -0.04456$.

7.7

38)a) $\Gamma\,(1) = \int_0^\infty e^{-t}dt = \lim_{b\to\infty} -e^{-t}\big|_0^b = 1$

$\Gamma\,(2) = \int_0^\infty te^{-t}dt$. If we integrate by parts using $u = t$, $dv = e^{-t}dt$ (so $du = dt$, $v = -e^{-t}$) we get

$$\begin{aligned} \Gamma\,(2) &= \lim_{b\to\infty} -te^{-t}\big|_0^b + \int_0^\infty e^{-t}dt \\ &= \lim_{b\to\infty} \frac{-b}{e^b} + \Gamma\,(1) \\ &= 1 \end{aligned}$$

where we used L'Hopital's rule to evaluate the limit.

b) We wish to show

$$\begin{aligned} \Gamma\,(n+1) &= n\Gamma\,(n) \\ \int_0^\infty t^n e^{-t}dt &= n\int_0^\infty t^{n-1}e^{-t}dt \end{aligned}$$

So let's integrate $\Gamma\,(n+1)$ by parts using $u = t^n$, $dv = e^{-t}dt$ (so $du = nt^{n-1}dt$ and $v = -e^{-t}$):

$$\begin{aligned} \int_0^\infty t^n e^{-t}dt &= \lim_{b\to\infty}\left(-b^n e^{-b}\right) + \int_0^\infty nt^{n-1}e^{-t}dt \\ &= 0 + n\Gamma\,(n)\,. \end{aligned}$$

c) For the first few positive integers, we see that $\Gamma\,(1) = 1$, $\Gamma\,(2) = 1$, $\Gamma\,(3) = 2\Gamma\,(1) = 2$, $\Gamma\,(4) = 3\cdot 2$, $\Gamma\,(5) = 4\cdot 3\cdot 2$ and easily see that $\Gamma\,(n) = (n-1)!$.

40)The definition of energy on page 329 is

$$E = \int_a^b \frac{kq_1q_2}{r^2}dr$$

where E is the energy required to separate two particles of charges q_1 and q_2 (actually these are the magnitudes, or absolute values, of the charges, so they are always positive) which are originally a distance a apart to a distance b apart. $k = 9 \cdot 10^9$ is a constant (we give this for E measured in joules, q_i measured in coulombs, and distances a and b measured in meters).

To move two 1 coulomb charges from one meter to infinitely far apart, we have

$$\begin{aligned}
E &= \int_1^\infty \frac{k\,(1)\,(1)}{r^2}dr \\
&= k \lim_{b\to\infty} \int_1^b \frac{1}{r^2}dr \\
&= k \lim_{b\to\infty} \left(-\frac{1}{r}\right)\Bigg|_1^b \\
&= k \left(\lim_{b\to\infty}\left(-\frac{1}{b}\right) + 1\right) \\
&= k \\
&= 9 \cdot 10^9 joules
\end{aligned}$$