**MATH 323 Section 2**

# $\mathcal{TEST}$ 3

*March* $22^{nd}$, *2013*

**Your Name:** ______________________________________________

**Directions:**

a. You may NOT use your book or your notes or a calculator.
b. Please ask for extra scrap paper if needed.
c. Good Luck!

## Score:

| | |
|---|---|
| 1. | ____________ |
| 2. | ____________ |
| 3. | ____________ |
| 4. | ____________ |
| 5. | ____________ |
| Total | ____________ |

**1. (10pts)** Let $f : A \to B$ be a function. State what it means for $f$ to be injective and what it means for $f$ to be surjective.

**2. (22pts)** Consider the relation on $\mathbb{N}$ given by $aRb$ if there exists $k \in \mathbb{Z}$ such that $\frac{a}{b} = 2^k$.

**a. (12pts)** Show this is an equivalence relation.

**b. (10pts)** Give an example of two different equivalence classes (that is, find $x, y \in \mathbb{N}$ such that $E_x \neq E_y$, where $E_x$ and $E_y$ are the equivalence classes of $x$ and $y$, respectively).

**3. (22pts)** Let $f : A \to B$ and $g : B \to C$ be functions.

**a. (12pts)** Show that if $g \circ f$ is injective, then $f$ is injective.

**b. (10pts)** Give an example of functions $f$ and $g$ such that $g \circ f$ is injective but $g$ is not injective. Justify your answer.

**4. (24pts)** Let $f : A \to B$ be a function and let $S, T \subseteq A$ and $U, V \subseteq B$.

**a. (12pts)** Give a counterexample to the statement: if $f(S) \subseteq f(T)$, then $S \subseteq T$.

**b. (12pts)** Prove that if $U \subseteq V$, then $f^{-1}(U) \subseteq f^{-1}(V)$.

**5. (22pts)** Consider the set of numbers $S = \left\{ \frac{a}{2} + \frac{b}{3} : a, b \in \mathbb{Z} \right\}$.

**a. (7pts)** Show that $\mathbb{Z} \subseteq S$.

**b. (15pts)** Show that the set $S$ is countable.

**Extra Credit (10pts): you may do ONLY ONE of the following problems:**

**a.** Find a bijection $g$ between the set of even integers $E = \{2n : n \in \mathbb{Z}\}$ and the set of positive powers of 3, $P_3 = \{3^m : m \in \mathbb{N}\}$. Show it is a bijection.

**b.** Show $\bigcup_{x \in (0,1)} \left[x, \frac{1}{x}\right] = (0, \infty)$