Chapter Check for Chapters 3 and 4

November 3, 2015

1. Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & -1 & 3 & 1 \\ 2 & 1 & 2 & -1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{pmatrix}.$$

a. Using row and column operations, find invertible matrices P and Q such that PAQ is of the block form

$$\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

for a certain size identity matrix I where the other matrices have all zero entries.

b. Put A in reduced row echelon form.

c. Use the reduced row echelon form of A to find a collection of columns of A that form a basis for the column space.

2. a. Compute the determinant of the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 6 & 12 & 9 \\ 3 & 6 & 10 & 15 \\ 4 & 8 & 12 & 14 \end{pmatrix}$$

by using row operations (Hint: Recall the determinant of an upper triangular matrix).

b. Recall a matrix $A \in F^{n\times n}$ is skew-symmetric if $A^T = -A$. If the field is not of characteristic 2, use the determinant to show that A has rank less than n if n is odd.

3. (Comprehensive/graduate option only) Let $A \in F^{m\times n}$ and $B \in F^{n\times m}$. Show that $\det(I_m + AB) = \det(I_n + BA)$ by showing that

$$\begin{pmatrix} I_m & -A \\ B & I_n \end{pmatrix} = \begin{pmatrix} I_m & 0 \\ B & I_n \end{pmatrix}\begin{pmatrix} I_m & -A \\ 0 & I_n + AB \end{pmatrix} = \begin{pmatrix} I_m + BA & -A \\ 0 & I_n \end{pmatrix}\begin{pmatrix} I_m & 0 \\ B & I_n \end{pmatrix}$$

and using properties of determinants. [This is called Sylvester's determinant theorem.]