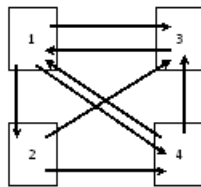# Introduction to PageRank

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Consider the following web.

1) Give a ranking of the pages in the web where x_k equal the number of backlinks for page k, i.e., the number of links into page k.

2) Explain why the rankings of pages 1 and 4 should not be the same since page 1 has a ranking from a more important page than page 4 does.

3) Now let x_k be the ranking of page k given so that the ranking of any page is equal to the sum of the rankings from each page linking into it. Show that this gives a systems of equations

$$x_1 = x_3 + x_4$$
$$x_2 = x_1$$
$$x_3 = x_1 + x_2 + x_4$$
$$x_4 = x_1 + x_2.$$

Now show that this system has no solutions! Hint: solve the last 3 equations and then show it is not consistent with the first equation.

4) Now let's change the ranking so that each page gets a single vote that is divided equally among its outlinks, and the ranking x_k is equal to the sum of

the votes times the rankings of all backlinks. The system is now

$$x_1 = x_3 + \frac{1}{2}x_4$$

$$x_2 = \frac{1}{3}x_1$$

$$x_3 = \frac{1}{3}x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_4$$

$$x_4 = \frac{1}{3}x_1 + \frac{1}{2}x_2.$$

Show that this system can be solved to get $x_1 = 1$, $x_2 = \frac{1}{3}$, $x_3 = \frac{3}{4}$, $x_4 = \frac{1}{2}$.

Notice that x_1 has the highest ranking! This is because x_3 threw its whole vote to x_1 and so that even though x_3 got votes from three different sites, they still do not total as much as what x_1 gets. Note, usually we will rescale so that the sum is equal to 1, and so we get

$$x_1 = \frac{12}{31}, \quad x_2 = \frac{4}{31}, \quad x_3 = \frac{9}{31}, \quad x_4 = \frac{6}{31}.$$