Math 537B Homework 2

February 6, 2006

1) a) Let B be a symmetric bilinear form on an inner product space (V, g), i.e.

$$B(x, y) = B(y, x).$$

Consider an orthonormal basis $e_1, \ldots, e_n$ of V so that if $x = x^i e_i$ then

$$B(x, x) = \sum_{i=1}^{n} \lambda_i \left(x^i\right)^2$$

for some real λ_i. Show that

$$\frac{1}{\omega_{n-1}} \int_{S^{n-1}} B(x, x)\, dS^{n-1} = \frac{1}{n} \sum \lambda_i$$

where $S^{n-1} = \partial B^n$ is the unit sphere, dS^{n-1} is the standard measure on the unit sphere, and ω_{n-1} is the $(n-1)$-dimensional volume of S^{n-1}.

b) Show that

$$S(p) = \frac{1}{\omega_{n-1}} \int_{S^{n-1}} Rc(X, X)\ dS^{n-1}(X)$$

where $S^{n-1} \subset T_pM$ is the unit sphere in T_pM.

c) Show that

$$Rc(X, X) = \frac{1}{\omega_{n-2}} \int_{S^{n-2}} K(X, Y)\ dS^{n-2}(Y)$$

where S^{n-2} is the unit sphere in T_pM orthogonal to X.

2) Exercise 8.5 number 1 on p. 141.

3) Problem 8-7