3) Preview of next homework: don’t turn in, but start thinking about the Taylor expansion of the volume of a geodesic ball of radius $r$, $V(r)$. It will require computing a Taylor series for $\sqrt{\det g_{ij}}$ which will use the following two facts which you should become familiar with. For a matrix valued function $g_{ij}(t)$,

$$\frac{d}{dt} \log \det g = g^{ij} \frac{d}{dt} g_{ij}$$

$$\frac{d}{dt} g^{ij} = -g^{ik} \left( \frac{d}{dt} g_{kj} \right) g^{lj}$$

where $g^{ij}$ is the inverse matrix and in the first line $g$ represents the matrix $g_{ij}$.