

Revitalizing College Algebra

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Diverse Goals for College Algebra

- Quantitative Literacy/Liberal Arts Mathematics
 - Meeting Requirements
 - Terminal Course
- Social Sciences/Business
 - Next course applied calculus
- Natural Sciences
 - Next course trig or precalculus
 - Intend to take mainstream calculus
- Preservice teachers
 - Middle school
 - High school

- What have we missed?
- Who is your audience and what are your goals?

Underlying subject matter for college algebra

- Arithmetic is about numbers and operations
- Algebra represents calculations with numbers and operations abstractly using letters
 - expressions, equivalent expressions give the same output
 - equations, equivalent equations have the same solutions
- Functions form a higher level of abstraction: letters are used to stand for relationships defined by
 - Expressions
 - Graphs
 - Tables
 - Words

- Which parts of this fit the goals of your course?
- Do you want to focus on “the logical process” or “the algebra”? Is there a conflict?

What is Algebraic Literacy?

- Interpreting algebraic form.
- Symbolic fluency.
- Algebraic foresight: anticipating the results of a calculation.
- Strategic manipulation: knowing which form to choose when.

Difference between expressions and equations

- An expression is built up out of variables, constants, and the operations of arithmetic. It has no equals sign.
- An equation is a statement of equality between two expressions.
- The confusion between equations and expressions causes students to do things like spontaneously remove the 2s in

$$\frac{x}{2} - \frac{3}{2}$$

Interpreting Algebraic Form

$$P\left(1 + \frac{r}{12}\right)^{12n}$$

$$L_0 \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

$$\frac{\sigma}{\sqrt{n}}$$

Interpreting Algebraic Form

A street vendor of t-shirts finds that if the price of a t-shirt is set at $\$p$, the profit from a week's sales is

$$(p - 6)(900 - 15p).$$

Which form of this expression shows most clearly the maximum profit and the price that gives that maximum?

- A. $(p - 6)(900 - 15p)$
- B. $-15(p - 33)^2 + 10935$
- C. $-15(p - 6)(p - 60)$
- D. $-15p^2 + 990p - 5400$

Answer: B. Since $(p - 33)^2$ is a square, it is always positive or zero, and it is only zero when $p = 33$. In the expression for the profit, a negative multiple of this square is added to 10,935. Thus the maximum profit is \$10,935, and the price which gives that profit is \$33.

Do the following equations have solutions?

$$\frac{2x + 5}{3x + 7} = 1$$

$$\frac{3x + 5}{3x + 7} = 1$$

Algebraic Foresight

Suppose P and Q give the sizes of two different animal populations, where $Q > P$. Say which of the given pair of expressions is larger. Briefly explain your reasoning in terms of the two populations.

$$P + Q \quad \text{and} \quad 2P$$

$$\frac{P}{P + Q} \quad \text{and} \quad \frac{P + Q}{2}$$

$$(Q - P)/2 \quad \text{and} \quad Q - P/2$$

$$P + 50t \quad \text{and} \quad Q + 50t$$

Strategic Manipulation

If R_1 is fixed, is

$$\frac{R_1 + R_2}{R_1 R_2}$$

increasing or decreasing as a function of R_2 ?

$$\frac{1}{R_1} + \frac{1}{R_2}$$

Strategic Manipulation

After a container of ice-cream has been sitting in a room for t minutes, its temperature in degrees Fahrenheit is

$$a - b2^{-t} + b,$$

where a and b are positive constants. Write this expression in a form that

- shows that the temperature is always greater than a .
 $a + b(1 - 2^{-t})$
- shows that the temperature is always less than $a + b$.
 $a + b - b2^{-t}$

Positive or Negative?

$$-(b - 1)^2$$

$$c\sqrt{-c}$$

$$(1 + 2^r)^2 - 1$$

Equivalent Expressions

1

- (a) Write an algebraic expression representing each of the following operations on a number b :

“Multiply by 0.4”

“Divide by five-halves”

- (b) Are these expressions equivalent? What does this tell you?

2

To convert from miles to kilometers, Abby takes the number of miles, doubles it, then subtracts 20% from the result. Renato first divides the number of miles by 5, and then multiplies the result by 8.

- (a) Write an algebraic expression for each method.
(b) Use your answer to part (a) to decide if the two methods give the same answer.

What is an Equation?

In the following problems, the solution to the equation depends on the constant a . Assuming a is positive, what is the effect of increasing a on the value of the solution? Does the solution increase, decrease, or remain unchanged? Give a reason for your answer that can be understood without solving the equation.

- 1 $x - a = 0$. *Increases. The larger a is, the larger x must be to give 0.*
- 2 $ax = 1$. *Decreases. The larger a is, the smaller x must be to give a product of 1.*
- 3 $ax = a$. *Remains unchanged. As a changes, the two sides of the equation change together and remain equal.*
- 4 $\frac{x}{a} = 1$. *Increases. The larger a is, the larger x must be to give a ratio of 1.*

Writing Equations

A ball thrown vertically upwards at a speed of v ft/sec rises a distance d feet in t seconds, given by $d = 6 + vt - 16t^2$. Write an equation whose solution is the given value.

- The time it takes a ball thrown at a speed of 88 ft/sec to rise 20 feet.
- The velocity with which the ball must be thrown to rise 30 feet in 2 seconds.