Pythagorean Triples

$s \rightarrow$ $r \downarrow$	1	2	3	4	5	6	7
2	(3,4,5)						
3	(8,6,10)	(5,12,13)					
4	(15,8,17)	(12, 16,20)	(7,24,25)				
5	(24,10,26)	(21,20,29)	(16,30,34)	(9,40,41)			
6	(35,12,37)	(32,24,40)	(27,36,45)	(20,48,52)	(11,60,61)		
7	(48,14,50)	(45,28,53)	(40,42,58)	(33,56,65)	(24,70,74)	(13,84,85)	
8	(63,16,65)	(60,32,68)	(55,48,73)	(48,64,80)	(39,80,89)	(28,96,100)	(15,112,113)

Triples (a, b, c) where $c^2 = a^2 + b^2$:

$$a + bi = (r + si)^2 = (r^2 - s^2) + (2rs)i$$

$$a = r^2 - s^2,\ b = 2rs,\ c = \mathrm{N}(r + si) = r^2 + s^2.$$

Eisenstein Triples

$s \rightarrow$ / $r \downarrow$	1	2	3	4	5	6	7	8	9
2	(3,3,3)								
3	(8,5,7)	(5,8,7)							
4	(15,7,13)	(12,12,12)	(7,15,13)						
5	(24,9,21)	(21,16,19)	(16,21,19)	(9,24,21)					
6	(35,11,31)	(32,20,28)	(27,27,27)	(20,32,28)	(11,35,31)				
7	(48,13,43)	(45,24,39)	(40,33,37)	(33,40,37)	(24,45,39)	(13,48,43)			
8	(63,15,57)	(60,28,52)	(55,39,49)	(48,48,48)	(39,55,49)	(28,60,52)	(15,63,57)		
9	(80,17,73)	(77,32,67)	(72,45,63)	(65,56,61)	(56,65,61)	(45,72,63)	(32,77,67)	(17,80,73)	
10	(99,19,91)	(96,36, 84)	(91,51,79)	(84,64,76)	(75,75,75)	(64,84,76)	(51,91,79)	(36,96,84)	(19,99,91)

Triples (a, b, c) where $c^2 = a^2 + b^2 - ab$:

$$a + b\omega = (r + s\omega)^2 = (r^2 - s^2) + (2rs - s^2)\omega \qquad \omega = \tfrac{-1+i\sqrt{3}}{2}$$

$$a = r^2 - s^2,\ b = 2rs - s^2,\ c = \mathrm{N}(r + s\omega) = r^2 + s^2 - rs.$$

Nice Cubics

$s \rightarrow$ / $r \downarrow$	1	2	3	4	5	6
2	$54 - 27x + x^3$	$-128 - 48x + x^3$				
3	$286 - 147x + x^3$	$286 - 147x + x^3$	$-1458 - 243x + x^3$			
4	$-506 - 507x + x^3$	$3456 - 432x + x^3$	$-506 - 507x + x^3$	$-8192 - 768x + x^3$		
5	$-7722 - 1323x + x^3$	$10582 - 1083x + x^3$	$10582 - 1083x + x^3$	$-7722 - 1323x + x^3$	$-31250 - 1875x + x^3$	
6	$-35282 - 2883x + x^3$	$18304 - 2352x + x^3$	$39366 - 2187x + x^3$	$18304 - 2352x + x^3$	$-35282 - 2883x + x^3$	$-93312 - 3888x + x^3$

$g(x) = x^3 - 3q^2x + d$ where

$$(1 + 2\omega)(r + s\omega)^2 = m + n\omega$$

$$3q^2 = \mathrm{N}\left(m + n\omega\right)$$

$$d = m\left(m^2 - 3q^2\right)$$

More Nice Cubics

$s \rightarrow$ $r \downarrow$	1	2	3	4
2	$28 - 24\,x + 3\,x^2 + x^3$	$-175 - 45\,x + 3\,x^2 + x^3$		
3	$140 - 144\,x + 3\,x^2 + x^3$	$140 - 144\,x + 3\,x^2 + x^3$	$-1700 - 240\,x + 3\,x^2 + x^3$	
4	$-1012 - 504\,x + 3\,x^2 + x^3$	$3025 - 429\,x + 3\,x^2 + x^3$	$-1012 - 504\,x + 3\,x^2 + x^3$	$-8959 - 765\,x + 3\,x^2 + x^3$

These are the translates $g(x+1)$ of some of the gs from the previous page ($2 \le r \le 4$, $1 \le s \le r$).