Linear Independence of Vectors: How To Really Digest It?

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Maria Teodora raised the following problem she has experienced:

My students have difficulties understanding the idea of linear independence.

Our approach to solve this problem is as follows. First, we should give a good geometric intuition of the meaning of linear combinations and linear dependence/independence. Draw vectors in the plane with various possibilities etc Hopefully this should attach some meaning to the definition.

Next, we design some true/false questions trying to address potential problems and misunderstandings. Here are some possible examples:

TRUE/FALSE QUESTIONS:

- The only way to write $(0,0)$ as a linear combination of $(1,2)$ and $(2,4)$ is
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- $(1,2)$ and $(2,4)$ are linearly independent.
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- Can you write $(0,0)$ as a linear combination of $(1,2)$ and $(2,4)$ in 5 different ways.
- Are $(0,0)$ and $(2,3)$ linearly independent?
- Can three vectors in $\mathbb{R}^2$ be linearly independent?
- $$(6,8) = (3*2, 4*2)$$
therefore $(6,8)$ is a scalar multiple of $(2,2)$.
- If $5v_1 = -6v_2$, can v_1, v_2 be linearly independent?