



INSTITUTE FOR
MATHEMATICS &
EDUCATION

VIGRE
Workshop on
Teaching Proof

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The problem (from Barbara Shipman)

Some of my students in Analysis I still have trouble with properly declaring the existence of, stating the meaning of, or defining objects used in the proof. For example, I have seen mistakes such as the following:

- 1 Given that a function g from X to Y is surjective, a student might write “If y is in Y , then $g(x) = y$ ”. The reader is left to wonder what x is.
- 2 Sometimes a student will define a function h from \mathbb{N} to \mathbb{N} as something like $h(x) = f(x/2)$ if x is even and $h(x) = g((x + 1)/2)$ if x is odd, without telling the reader what f and g are.
- 3 A student may start a proof by writing “For all $\epsilon > 0$, there exists $\delta > 0$ such that ...” and immediately thereafter consider the interval $(f(p) - \epsilon, f(p) + \epsilon)$, without having chosen any particular ϵ .



A possible solution

This series of problems is intended to lead students to grapple with the different contexts in which the same variable is used.

- 1 Find a value of x which minimizes $f(x) = x^2 - 2x + 2$.
- 2 Prove that $f(x)$ is greater than zero for all values of x .
- 3 Prove that there exists an x such that $f(x)$ is twice the minimum value of $f(x)$.
- 4 Prove that there does not exist an x such that $f(x) = -1$.
- 5 Prove that the set of matrices of the form

$$\begin{pmatrix} x & y \\ 0 & 1 \end{pmatrix}, \quad x, y \in \mathbb{R}$$

is closed under matrix multiplication.

- 6 Show that the multiplication in (5) is not commutative.