I. VOCABULARY AND LANGUAGE

The following explains, defines, or lists some of the words that may be used by authors of the Olympiad problems.

1. **Basic Terms**
   Sum, difference, product, quotient, ratio, square of a number, factors of a number.

2. **Reading**
   Read “1 + 2 + 3 + …” as “one plus two plus three and so forth”.
   Read “1 + 2 + 3 + … + 10” as “one plus two plus three and so forth up to ten.”
   In reading numbers, use “and” only before a fraction or decimal fraction.
   Examples: Read 305 3/4 as “three hundred five and one-fourth”
   Read 1001.2 as “one thousand one and two-tenths”.

3. **Standard Form of a Number**
   A digit is any one of the ten numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Combinations of digits assigned place values are used to write all numbers. A number may be described by the number of digits it contains: 358 is a three-digit number. The “lead-digit” (leftmost digit) of a number is not counted as a digit if it is 0: 0358 is a three-digit number. **Terminal zeros** of a number are the zeros to the right of all nonzero digits: 30,500 has two terminal zeros.

   The standard form of a number refers to the form in which we usually write numbers (also called Hindu-Arabic numerals or positional notation). The expanded form of a number is the sum which is associated with the standard form.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Expanded Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>358</td>
<td>3×100 + 5×10 + 8×1, or 300 + 50 + 8, or 3×10² + 5×10 + 8×1</td>
</tr>
</tbody>
</table>

4. **Sets of Numbers**
   **Whole Numbers** = {0, 1, 2, 3, ...}
   **Natural or Counting Numbers** = {1, 2, 3, ...}
   In the Division E negative numbers will not be used in Olympiad problems.
   **Consecutive Numbers** are natural numbers that differ by 1, such as 83, 84, 85, 86 and 87.
   **Consecutive Even Numbers** are multiples of 2 that differ by 2, such as 36, 38, 40, and 42.
   **Consecutive Odd Numbers** are nonmultiples of 2 that differ by 2, such as 57, 59, 61 and 63.

5. **Divisibility**
   Let A and B be natural numbers. Then A is divisible by B if B divides A with zero remainder (or \(A/B\) is equal to a natural number). In such instances B is called a **factor** of A, and A is called a **multiple** of B.
NUMBER THEORY

a. A prime number is a natural number which has exactly two different factors, namely itself and 1. Note that 1 is not a prime number. Examples: 2, 3, 5, 7, 11, 13, . . .

b. A composite number is a natural number which has more than two different factors, namely 1, itself, and at least one other factor. There are 3 categories of natural numbers: prime, composite and 1. Examples: 4, 6, 8, 9, 10, 12, . . .

c. A number is factored completely when it is expressed as a product of prime numbers.
   Example: 144 = 2×2×2×2×3×3. It may also be written as 144 = 2^4×3^2.

d. The Greatest Common Factor (GCF) of two natural numbers is the largest natural number that divides each of the two given numbers with zero remainder. Example: GCF(12,18) = 6.

e. If the GCF of two numbers is 1, then we say the numbers are relatively prime or co-prime.

f. The Least Common Multiple (LCM) of two natural numbers is the smallest number that each of the given numbers divides with zero remainder. Example: LCM(12,18) = 36.

g. Order of Operations. When computing the value of expressions involving two or more operations, the following priorities must be observed from left to right:
   1) do operations in parentheses, braces, or brackets,
   2) do multiplication and division from left to right, and then
   3) do addition and subtraction from left to right.
   Example: 
   \[
   \begin{align*}
   &\quad 3 + 4 \times 5 - 8 \div (9 - 7) \\
   &= 3 + 4 \times 5 - 8 \div 2 \\
   &= 3 + 20 - 4 \\
   &= 19
   \end{align*}
   \]

7. FRACTIONS

a. A common (or simple) fraction is a fraction in the form \( \frac{a}{b} \) where \( a \) and \( b \) are whole numbers and \( b \neq 0 \).

b. A unit fraction is a common fraction with numerator 1.

c. A proper fraction is a common fraction in which \( a < b \). Its value is between 0 and 1.

d. An improper fraction is a common fraction in which \( a \geq b \). Its value is 1 or greater than 1.

e. A complex fraction is a fraction whose numerator or denominator contains a fraction.
   Examples: \( \frac{2/3}{5}, \frac{7}{3}, \frac{3 - 1/3}{3 + 1/3} \)

f. The fraction \( \frac{a}{b} \) is simplified if \( a \) and \( b \) have no common factor other than 1 [GCF(a,b) = 1].

8. TIME

Students should know the meaning of century and years spanned in a particular century. For example, the 19th century refers to the 100-year period preceding the year 1901; that is, the years from 1801 through 1900, inclusive.

9. AVERAGE

The average of a set of \( N \) numbers is the sum of the set of the \( N \) numbers divided by \( N \).
10. Geometry
   a. Angle: degree-measure.
   b. Kinds of angles: acute, right, obtuse, straight, reflex.
   c. Polygons:
      Triangles: acute, right, obtuse, scalene, isosceles, equilateral.
      note: an equilateral triangle is isosceles with all sides equal.
      Quadrilaterals: parallelogram, rectangle, square, trapezoid, rhombus.
      note: a square is a rectangle with equal sides.
      Others: pentagon, hexagon, octagon, decagon, dodecagon, icosagon.
      Area: the number of unit squares contained in the interior of a region.
      Perimeter: the number of unit lengths in the boundary of a plane figure.
      Circumference: the perimeter of a circular region.
      Congruent figures: two or more plane figures whose corresponding sides and angles
      have the same measure.
      Similar figures: two or more plane figures whose size may be different but whose shape
      is the same.

II. Skills

1. Computation
   The tools of arithmetic are needed for problem-solving. Competency in the basic operations
   on whole numbers, fractions, and decimals is essential for success in problem solving at the
   elementary and higher levels. If needed, parts of practice sessions should be used to introduce and
   develop the arithmetic skills which are associated with all problem solving.

2. Answers
   Unless otherwise specified in a problem, equivalent numbers or expressions should be accepted.
   For example, 3½, 7/2, and 3.5 are equivalent.

   Units of measure generally are not required in answers but must be correct if given in an an-
   swer. Measures of area are usually written as square units, sq. units, or units². For example,
   square centimeters may be abbreviated as sq cm, or cm x cm, or cm².

   After reading a problem, a wise procedure is to indicate the nature of the answer at the bottom
   of a worksheet before starting the work necessary for solution. Examples: “A = __, B = __”;
   “The largest number is __”. Another worthwhile device in practice sessions is to require the
   student to write the answer in a simple declarative sentence. Example: “The average speed is 54
   miles per hour.” This device usually causes the student to reread the problem.

3. Measurement
   The student should be familiar with units of measurement for time, length, area, and weight in
   English and metric systems.

   Conversions from one unit to another within systems:
   time: seconds to minutes to hours; hours to minutes to seconds
   length: inches to feet to yards; yards to feet to inches
   area: in² to ft² to yds²; yds² to ft² to in²
   weight: ounces to pounds; pounds to ounces.
III. SOME USEFUL THEOREMS

1. If a number is divisible by $2^n$, then the number formed by the last $n$ digits of the given number is also divisible by $2^n$; and conversely.

   Example: 7,292,536 is divisible by 2 (or $2^1$) because 6 is divisible by 2.
   Example: 7,292,536 is divisible by 4 (or $2^2$) because 36 is divisible by 4.
   Example: 7,292,536 is divisible by 8 (or $2^3$) because 536 is divisible by 8.

2. If the sum of the digits of a number is divisible by 9, then the number is divisible by 9.
   If the sum of the digits of a number is divisible by 3, then the number is divisible by 3.

   Example: 658,773 is divisible by 9 because $6+5+8+7+7+3 = 36$ which is a multiple of 9.
   Example: 323,745 is divisible by 3 because $3+2+3+7+4+5 = 24$ which is a multiple of 3.

3. A number is divisible by 11 if the difference between the sum of the odd-place digits and the sum of the even-place digits is 0 or a multiple of 11.

   Example: 90,728 is divisible by 11 because $(9+7+8)-(0+2) = 24 - 2 = 22$ which is a multiple of 11.

4. If A and B are natural numbers, then:
   (i) GCF(A,B) × LCM(A,B) = A × B.
   (ii) LCM(A,B) = (A × B) ÷ GCF(A,B).
   (iii) GCF(A,B) = (A × B) ÷ LCM(A,B).

   Example: If A = 9 and B = 12: GCF(9,12) = 3, LCM(9,12) = 36, A × B = 9 × 12 = 108.
   Then: (i) $3 \times 36 = 108$; (ii) $108 ÷ 3 = 36$; (iii) $108 ÷ 36 = 3$.

5. If $p$ represents a prime number, then $p^n$ has $n+1$ factors. Example: $2 \times 2 \times 2 = 2^3$ has 6 factors which are 1, 2, 2², 2×2, 2×2×2, 2×2×2×2. In exponential form, the factors are: 1, 2, 2², 2³, 2⁴, and 2⁵. In standard form, the factors are: 1, 2, 4, 8, 16, and 32. Notice that the factors of $2^5$ include 1 and $2^5$ itself.

Problem: how many factors does 72 have? $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$. Since $2^3$ has 4 factors and $3^2$ has 3 factors, 72 has $4 \times 3 = 12$ factors. The factors may be obtained by multiplying any one of the factors of $2^3$ by any one of the factors of $3^2$: $(1, 2, 2^2, 2^3) \times (1, 3, 3^2)$. Written in order, the 12 factors are: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72.

IV. SOME STRATEGIES FOR PROBLEM SOLVING

- Draw a picture or diagram
- Make an organized list
- Solve a simpler problem
- Work backward
- Find a pattern
- Make a table
- Guess, check and revise
- Use reasoning (logic)

Thorough discussions of these and many other useful topics may be found in Creative Problem Solving in School Mathematics and Math Olympiad Contest Problems for Elementary and Middle Schools.