Measuring Income Inequality in Elementary and High School

Charles Collingwood, Cassidy Larkin, Victor Piercey, and Jordan Schettler, G-TEAMS, University of Arizona
Who are We?

Charles Collingwood: Stats, Precalculus, and AP Calculus Teacher at Rincon High School.

Jordan Schettler: Math Ph.D. Student at University of Arizona.
Who are We?

**Cassidy Larkin:** Fifth Grade Teacher at McCartney Ranch Elementary School

**Victor Piercey:** Math Ph.D. Student at University of Arizona.
- GK-12 program housed in the UA Math Department.

- G-TEAMS stands for Graduate Students and Teachers Engaging in Mathematical Sciences.
McCartney Ranch Elementary
McCartney Ranch Elementary

- Located in Casa Grande, AZ.
McCartney Ranch Elementary

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- 1 of 9 elementary schools in the District.
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- High population of English Language Learners.
- Increasing population of “exceeds” math students.
Located in Casa Grande, AZ.

1 of 9 elementary schools in the District.

High population of English Language Learners.

Increasing population of “exceeds” math students.

Title I school with about 650 students, about 70% of students are receiving free/reduce lunch rates.
Income Inequality Data Analysis

- Each table has a piece of paper with some data and a sheet of graph paper.

- Make a graph (bar, line, circle, etc.) to represent this data.

- Identify any features or trends your graph illustrates.
Using Excel
In the computer lab, students learned rudiments of Excel.
In the computer lab, students learned rudiments of Excel.

For example, they:
Using Excel

- In the computer lab, students learned rudiments of Excel.
- For example, they:
  (i) entered data;
In the computer lab, students learned rudiments of Excel.

For example, they:
(i) entered data;
(ii) used auto-sum;
In the computer lab, students learned rudiments of Excel.

For example, they:
(i) entered data;
(ii) used auto-sum;
(iii) entered formulas; and
In the computer lab, students learned rudiments of Excel.

For example, they:

(i) entered data;
(ii) used auto-sum;
(iii) entered formulas; and
(iv) made various types of graphs.
Is $20,000 per year a lot?

Fifth grade students analyzed a monthly budget for local expenses.
Income Distribution in 2009: Sample One

<table>
<thead>
<tr>
<th>Income</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;29999</td>
<td>35,985</td>
</tr>
<tr>
<td>30000-59999</td>
<td>32,421</td>
</tr>
<tr>
<td>60000-89999</td>
<td>20,745</td>
</tr>
<tr>
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<td>12,179</td>
</tr>
<tr>
<td>120000-149999</td>
<td>6,495</td>
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<tr>
<td>150000-179999</td>
<td>3,799</td>
</tr>
<tr>
<td>&gt;180000</td>
<td>1,410</td>
</tr>
<tr>
<td>Mean</td>
<td>16,148</td>
</tr>
<tr>
<td>Median</td>
<td>12,179</td>
</tr>
</tbody>
</table>

U.S. Household Annual Income Distribution in 2009

![Chart showing income distribution in 2009]
Income Distribution in 2009: Sample Two

<table>
<thead>
<tr>
<th>income</th>
<th>number</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;29999</td>
<td>35985</td>
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<td>30000-59999</td>
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<tr>
<td>mean</td>
<td>16147.71</td>
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<tr>
<td>median</td>
<td>12179</td>
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</tbody>
</table>
Income Distribution 1969-2009: Sample One

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>4.10%</td>
<td>4.30%</td>
<td>4.10%</td>
<td>4%</td>
<td>3.80%</td>
<td>3.60%</td>
<td>3.60%</td>
<td>3.40%</td>
<td>3.40%</td>
</tr>
<tr>
<td>Top</td>
<td>16.60%</td>
<td>16.50%</td>
<td>16.90%</td>
<td>17.10%</td>
<td>18.90%</td>
<td>21.20%</td>
<td>21.50%</td>
<td>21.80%</td>
<td>21.70%</td>
</tr>
</tbody>
</table>

Income Distribution

- **Bottom 20%**
- **Top 5%**

Years

percent

0.00% 5.00% 10.00% 15.00% 20.00% 25.00%

0.00 1974 1984 1994 2004
Income Distribution 1969-2009: Sample Two

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>poverty</td>
<td>9.70%</td>
<td>8.80%</td>
<td>9.20%</td>
<td>11.60%</td>
<td>10.30%</td>
<td>11.60%</td>
<td>9.30%</td>
<td>10.20%</td>
<td>11.10%</td>
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</table>

![poverty rate graph]

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(Note: The graph shows the poverty rate trend from 1969 to 2009, with years 1969, 1979, 1989, 1999, and 2009 marked on the x-axis and poverty rates on the y-axis.)
Sample Two

<table>
<thead>
<tr>
<th></th>
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<td>11.10%</td>
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Poverty Rate

Percentage

0.00% 2.00% 4.00% 6.00% 8.00% 10.00% 12.00% 14.00%

Years


Poverty rate
Sample Responses to Questions
Question: Look at your graph and describe why some people say that “the rich are getting richer and the poor are getting poorer.”
Question: Look at your graph and describe why some people say that “the rich are getting richer and the poor are getting poorer.”

Response: Because the bottom goes down and the top goes up.
Question: Look at your graph and describe why some people say that “the rich are getting richer and the poor are getting poorer.”

Response: Because the bottom goes down and the top goes up.

Response: Because throughout the years the top 5 starts and is getting a larger increase than the bottom 20.
Sample Responses to Questions
Question: What do you think can be done in order to make this line graph (for poverty rates) decrease between 2009 and 2014?
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Response: I think we could help the homeless and hungry get money.
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Response: I think we could help the homeless and hungry get money.

Response: More people should leave the United States.
Standards
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- Collect, record, organize and display data.
Standards

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- Formulate and answer questions by interpreting displays of data.
Standards

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- Describe patterns of change.
Standards

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- Formulate and answer questions by interpreting displays of data.
- Describe patterns of change.
- Summarize mathematical information.
Reflections
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- The real-world application along with the use of real world data was a great motivator for students!
Reflections

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- Some students who had been previously unengaged became enthusiastic. The context led to student “buy-in”.

![Image of students engaged in activity]
Reflections

- The real-world application along with the use of real world data was a great motivator for students!
- Some students who had been previously unengaged became enthusiastic. The context led to student “buy-in”.
- When brought back to first principles and properly motivated, young students can understand fairly complicated concepts.
Reflections

- The real-world application along with the use of real world data was a great motivator for students!
- Some students who had been previously unengaged became enthusiastic. The context led to student “buy-in”.
- When brought back to first principles and properly motivated, young students can understand fairly complicated concepts.
- This type of activity is very difficult to fit within every-day teaching constraints.
Reflections (continued)
Reflections (continued)

- Students tended to not think about meaning after they finished making the graph.
Reflections (continued)

- Students tended to not think about meaning after they finished making the graph.
- Some students didn’t understand that they had to read their instructions in order to know what to do next.
Students tended to not think about meaning after they finished making the graph.

Some students didn’t understand that they had to read their instructions in order to know what to do next.

Some students had trouble understanding when to use their instructions and when to use their data sheet.
Rincon High School
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- 1 of 22 high schools in TUSD
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- Demographics: 48.5% Hispanic, 34.1% White, 9.0% African American, 6.7% Asian American, 1.3% Native American, 0.4% Multi
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- Demographics: 48.5% Hispanic, 34.1% White, 9.0% African American, 6.7% Asian American, 1.3% Native American, 0.4% Multi
- Significant % of free/reduced lunch
Mathematical Practices in the New Common Core Standards
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1. Make sense of problems and persevere in solving them.
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5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Math Achievement and Ethnicity: $\chi^2$-Independence Test

HIGHEST MATH COURSE TAKEN BY ETHNICITY

What’s the story at Rincon?

<table>
<thead>
<tr>
<th></th>
<th>Hispanic</th>
<th>Black</th>
<th>White</th>
<th>Asian</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>48 (58)</td>
<td>11 (15)</td>
<td>43 (40)</td>
<td>20 (8)</td>
</tr>
<tr>
<td>B</td>
<td>73 (79)</td>
<td>14 (21)</td>
<td>65 (55)</td>
<td>14 (11)</td>
</tr>
<tr>
<td>C</td>
<td>76 (96)</td>
<td>29 (25)</td>
<td>80 (66)</td>
<td>15 (13)</td>
</tr>
<tr>
<td>D</td>
<td>128 (116)</td>
<td>28 (31)</td>
<td>80 (81)</td>
<td>8 (16)</td>
</tr>
<tr>
<td>F</td>
<td>199 (175)</td>
<td>56 (46)</td>
<td>96 (122)</td>
<td>16 (25)</td>
</tr>
</tbody>
</table>

Observed vs. (Expected) Grades in Math, 2010

The test statistic is $= 52.83061161$....
The $p$-value is $= 0.00000044152365$.

Assuming a student’s grade in math is independent of their ethnicity, the $p$-value is the probability of getting a test statistic $\geq \Sigma$. Here the $p$-value is very small, so we should reject the assumption of independence. We also used this data to create frequency tables from each column. Then students computed mean, standard deviation, and histograms for GPA.
Line of perfect equality (y=x)
Lorenz curve for the United States (based on the 1998 income shares in Exhibit 1).
Interpolating Lorentz Curves

- Enter the data: STAT, 1
- Find a regression equation: STAT, ▶, 7
- Graph a regression equation: Y=, VARS, 5, ▶, ▶, 1

<table>
<thead>
<tr>
<th></th>
<th>1968</th>
<th>1992</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.2</td>
<td>0.042</td>
<td>0.038</td>
</tr>
<tr>
<td>0.4</td>
<td>0.155</td>
<td>0.132</td>
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<tr>
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<td>0.574</td>
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<tr>
<td>1.0</td>
<td>1.000</td>
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</tr>
</tbody>
</table>

US Cumulative Household Income Shares by Quintile
The Gini Coefficient $G$

- $G := \frac{A}{A + B}$

- If $L(x) = \text{Lorenz curve}$,
  \[ B = \int_0^1 L(x) \, dx \]

- Also, $A + B = 1/2$, so
  \[ G = 1 - 2B \]
Gini Index - Income Disparity since World War II

where 0 is perfect equality, and 100 is perfect inequality (i.e., one person has all the income)
Computing Gini Coefficients

- Enter the data: STAT, 1
- Find a regression equation: STAT, ▶, 7
- Graph a regression equation: Y=, VARS, 5, ▶, ▶, 1
- Find B: 2nd, CALC, 7
- Then $G = 1 - 2B$. 

<table>
<thead>
<tr>
<th>x</th>
<th>1968</th>
<th>1992</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
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</tbody>
</table>

US Cumulative Household Income Shares by Quintile
Point of Maximal Inequality (POMI)

Lorenz Curve for the United States in 1998

\[ y = x \]

\[ y = L(x) \]
Point of Maximal Inequality (POMI)

Lorenz Curve for the United States in 1998

$y = x$

$y = L(x)$

(0.651, 0.318) point of maximal inequality
The POMI is a min of $L(x) - x$
Tracking the POMI

- Subtract $x$ from a regression equation

<table>
<thead>
<tr>
<th>$x$</th>
<th>1968</th>
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<tr>
<td>0.0</td>
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- Find a minimum: $2$nd, CALC, 3

- Interpret the movement of this point.

US Cumulative Household Income Shares by Quintile
In most schools today, teachers have constraints. Curriculum calendars and scripted curricula do not leave students with the opportunity to have in depth exploration time. Are we producing tomorrow’s problem solvers?

What kinds of ideas do you have that could turn into a possible project that relates to real world problems, yet that have fidelity to school districts?

ELEMENTARY: LEFT SIDE OF ROOM.
MIDDLE AND HIGH: RIGHT SIDE.