Making Formulas to Solve Linear Systems

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1 Creating a formula

Formulae are plug-and-chug methods of finding an unknown quantity. For example, you know the formula to calculate the slope of a line:

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]  

However, how do you come up with formulae? Wouldn’t it be nice if there were a method easier than what you’re currently doing to solve systems of linear equations?

For example, let’s say we have two equations

\[ 2x + y = 3 \]  

\[ 3x + y = 5 \]

You know how to solve this. But let’s say we see lots of systems of equations, like \( \frac{15}{7}x + y = 13 \), \( \frac{13}{7}x + y = 10 \), etc. What similarities do all these equations share? All the y-coefficients are one. Perhaps we can make life simpler if we found a faster method of solving systems of linear equations when y has a coefficient of one.

To generalize, first let’s assume the coefficient of x can be anything, call it A. Let’s also call the constant C. So now we have

\[ Ax + y = C \]  

But wait, we need two equations to solve for this right? We can’t call both x coefficients A, otherwise that means they’re the same number. So, let’s modify it so that the x coefficients of the first equation are called \( A_1 \) and the second equation’s is \( A_2 \). Same thing with C.

\[ A_1x + y = C_1 \]  

\[ A_2x + y = C_2 \]

How would we solve for this equation? Well, cancel the y’s first. Subtract \( 5 \) from \( 6 \).

\[ (A_2 - A_1)x = C_2 - C_1 \]

\[ x = \frac{C_2 - C_1}{A_2 - A_1} \]

Putting this back in for x, we get

\[ A_2x + y = C_2 \]

\[ A_2 \cdot \frac{C_2 - C_1}{A_2 - A_1} + y = C_2 \]

\[ y = C_2 - A_2 \cdot \frac{C_2 - C_1}{A_2 - A_1} \]

Does this really work? Let’s try it.

\[ 2x + y = 3 \]
\[ 3x + y = 5 \quad (13) \]

\[ A_1 = 2, \ A_2 = 3, \ C_1 = 3, \ C_2 = 5. \]

\[ y = 5 - 3 \cdot \frac{5 - 3}{3 - 2} \quad (14) \]

\[ y = 5 - 6 \]

\[ y = -1 \quad (15) \]

\[ x = \frac{5 - 3}{3 - 2} \]

\[ x = \frac{2}{1} \quad (17) \]

\[ x = 2 \quad (18) \]

So, \( x = 2 \) and \( y = -1 \). Plugging those numbers back in confirms it.

Do the same for

\[ x + B_1 y = C_1 \quad (20) \]

\[ x + B_2 y = C_2 \quad (21) \]