Linear Regression with the Tax Table

September 30, 2009

In this activity, we will use linear regression on our calculators to estimate tax rates in two different income brackets. The data comes from the Internal Revenue Service 2008 form 1040 instructions. We assume the filing status of the taxpayer is “single.”

Before we begin, you should clear your lists.

1. Insert the following data into two lists in your calculator:

<table>
<thead>
<tr>
<th>Taxable Income</th>
<th>Tax Owed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1000</td>
<td>101</td>
</tr>
<tr>
<td>2000</td>
<td>201</td>
</tr>
<tr>
<td>3000</td>
<td>303</td>
</tr>
<tr>
<td>4000</td>
<td>403</td>
</tr>
<tr>
<td>5000</td>
<td>503</td>
</tr>
<tr>
<td>6000</td>
<td>603</td>
</tr>
<tr>
<td>7000</td>
<td>703</td>
</tr>
<tr>
<td>8000</td>
<td>803</td>
</tr>
</tbody>
</table>

2. Use the StatPlot menu to set your calculator to graph a scatterplot. Adjust your window appropriately and graph. Does it appear that there is a correlation between taxable income and taxes owed? Does it look linear?

3. Let’s perform linear regression. Hit the stat button, move the cursor to the right to highlight the “calc” menu, move the cursor down to “LinReg (ax + b)” and hit enter. Using the 2nd key, enter $L_1$, comma, $L_2$, comma. Then hit the vars button, move the cursor left to select “Y-VARS,” select function and hit enter, select $Y_1$ and hit enter. Hit enter again.

   (a) What is the slope? What does this tell you?
   (b) What is the intercept? What does this tell you?
   (c) Your calculator also reports $r^2$ and $r$. What are these values and what do they tell you?
   (d) Press the “graph” button. Your display will now show your data points and will plot the line described by your linear regression model. This line is called the “regression line.”

4. Use the model from your linear regression to predict the taxes owed for an income of 5600. The actual value is 563. How close were you?
5. Use the model from your linear regression to predict the taxes owed for incomes of 10,000, 50,000 and 90,000. When you are predicting the values for inputs outside of the range of regression, this is called **extrapolation**.

6. The actual taxes owed for 10,000 are 1,103, for 50,000 are 8,850 and for 90,000 are 19,185. What do you think accounts for the difference between the predicted values and the actual values?

7. Repeat steps one through three above for incomes between 90,000 and 98,000 given by the following table:

<table>
<thead>
<tr>
<th>Taxable Income</th>
<th>Tax Owed</th>
</tr>
</thead>
<tbody>
<tr>
<td>90,000</td>
<td>19,185</td>
</tr>
<tr>
<td>91,000</td>
<td>19,465</td>
</tr>
<tr>
<td>92,000</td>
<td>19,745</td>
</tr>
<tr>
<td>93,000</td>
<td>20,025</td>
</tr>
<tr>
<td>94,000</td>
<td>20,305</td>
</tr>
<tr>
<td>95,000</td>
<td>20,585</td>
</tr>
<tr>
<td>96,000</td>
<td>20,865</td>
</tr>
<tr>
<td>97,000</td>
<td>21,145</td>
</tr>
<tr>
<td>98,000</td>
<td>21,425</td>
</tr>
</tbody>
</table>

8. Use this model to predict the taxes owed for an income of 94,500. The actual value is 20,445. How close is your prediction?

9. Use this model to extrapolate the taxes owed for an income of 100,000. The actual value is 21,705. How close is your extrapolation?