Modeling Taxes

Algebra 5/Trig

Spring 2010

Instructions: There are none! This contains background information and some suggestions for your project. Feel free to make changes and ask different questions if you want, subject to your teacher’s approval.

1 Background

The federal income tax system is known as progressive taxation. The main characteristic of progressive taxation is that the higher your income is, the higher the tax rate you face. In the United States, this is achieved through the application of marginal tax rates. An example of such a system is as follows:

<table>
<thead>
<tr>
<th>Income Level (per year)</th>
<th>Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under $20,000</td>
<td>10%</td>
</tr>
<tr>
<td>Between $20,000 and $50,000 (inclusive)</td>
<td>$2,000 + 20% of income over $20,000</td>
</tr>
<tr>
<td>Over $50,000</td>
<td>$8,000 + 30% of income over $50,000</td>
</tr>
</tbody>
</table>

In other words, if you earn up to $20,000 per year, you pay 10% of your income in taxes. If you earn between $20,000 and $50,000, you pay 10% of the first $20,000 of income and 20% of the rest (the margin). Finally, if you make over $50,000 per year, you pay 10% of the first $20,000, then you pay 20% of the next $30,000 and then 30% of the remaining income. This can be written as a piecewise defined function. If \( x \) is the annual income of a taxpayer and \( y = f(x) \) is the amount of tax they owe, then we can write:

\[
f(x) = \left\{ \begin{array}{ll}
0.1x & , \quad x < 20,000 \\
2,000 + 0.2(x - 20,000) & , \quad 20,000 \leq x \leq 50,000 \\
8,000 + 0.3(x - 50,000) & , \quad x > 50,000
\end{array} \right.
\]

Each of these expressions can be simplified to obtain:

\[
f(x) = \left\{ \begin{array}{ll}
0.1x & , \quad x < 20,000 \\
0.2x - 2,000 & , \quad 20,000 \leq x \leq 50,000 \\
0.3x - 7,000 & , \quad x > 50,000
\end{array} \right.
\]

The regions of the domain used to define the function are called tax brackets or sometimes income brackets.

Note that in the United States we have different rate structures for different types of filers, such as
those who are single, those who are married and file jointly, etc. We will just focus on single filers, to keep things simple.

You may want to review piecewise-defined functions in section 2.2 and their graphs in section 2.3, linear functions in section 2.1, and transformations in section 2.4 in your textbook.

2  Some Questions

1. Find out the current marginal rate structure and write a function that models our tax system. Graph this function.

2. Several politicians talk about a “flat tax.” Look this up and describe the proposals with a mathematical function. Graph the function on the same set of axes you used to graph the current system. Who pays more under the flat tax proposal? Who pays less? Do you think this is fair? Do you think the flat tax proposal is good policy?

3. A deduction is an amount that is subtracted from income before applying the tax function. The current tax structure employs a standard deduction, for taxpayers who have no other deductions. Find the current standard deduction and rewrite your tax function to account for the standard deduction. How will this change the graph? For simplicity, you may just want to use the flat tax model for this, but you can certainly use the current system if you want.

4. A credit is an amount that is subtracted from the tax due that is the output of the tax function. Incorporate a $200 credit into your tax functions. How does this change the graph? Which is better for the taxpayer, a $200 deduction or a $200 credit?