Introduction
A graph is a collection of dots (vertices) and lines (edges). Each edge has a vertex at each end. The following games involve graphs.

Triangles
For this game, we begin with a complete graph. In a complete graph, every vertex is connected to every other vertex (not including itself) by an edge. In particular, we’ll be using the complete graph on six vertices:

![Complete graph on six vertices]

The rules:

- There are only two players. Each player needs a different colored writing utensil.
- The players take turns coloring in a single line in the complete graph on six vertices. The first player to form a triangle with their color wins.

Try playing the game on the graph above. A few more graphs are provided on the next page, but you can redraw it and play as many times as you’d like.
1. Is it possible for the game to end in a tie? Why or why not?

2. What are some strategies you notice? What does a winning position look like?
Chomp the graph

For this game, we begin with a simple graph. A simple graph has no loops (each edge has two different vertices at its endpoints) or multiple edges (for any two vertices, there is at most one edge between them).

Here are some examples of simple graphs:

![Simple graph examples](image)

The rules:

- Start with a simple graph
- Two players take turns. On your turn, you may remove one vertex and all of its incident (connected) edges, or you may remove one edge.
- The winner of the game is the one who removes the last vertex of the graph. (The loser is the player who can no longer remove any vertices or edges.)

Let’s start out with a linear graph:
Do you see any strategies for linear graphs?

Next try a tree. This is a graph without any cycles (no set of vertices is connected together in a chain). An example of a tree is the graph pictured on the left at the beginning of this section.

Do you see any strategies for tree graphs?

Next try a cycle. This is a graph with all of the vertices connected in a closed chain, like the graph pictured on the right at the beginning of this section.

Do you see any strategies for cycles?

Next try a complete graph (we used these in the first section for the triangle game). Do you see any strategies for complete graphs?
Kayles

The game Kayles was invented by Henry Dudeney in the early 1900s. It was originally played just on line graphs, but can be played on any type of graph.

The rules:

- Draw a simple graph. Make the vertices open circles instead of solid dots, since we will be coloring them in.
- There are two players. On each turn, a player can *either* shade in one circle, or shade in two connected circles.
- The player to shade in the last circle wins.

Try playing Kayles with a few of the same graphs we used in the previous games. Do you notice any strategies? Does the player who goes first or second have an advantage?