Introduction
Do you recognize any of these sequences? What should the next number be?

\[
\begin{align*}
4, & \ 18, \ 32, \ 50, \ 64, \ \ldots \ \\
1, & \ 1, \ 2, \ 3, \ 5, \ 8, \ 13, \ \ldots \ \\
2, & \ 6, \ 18, \ 54, \ 162, \ \ldots \ \\
30, & \ 28, \ 32, \ 30, \ 34, \ \ldots \ \\
16, & \ 17, \ 18, \ 18, \ 20, \ 19, \ \ldots \ \\
1, & \ 1, \ 2, \ 5, \ 12, \ \ldots
\end{align*}
\]

There many many different types of sequences. The online encyclopedia of integer sequences now lists about 200,000 different sequences that people have studied. (oeis.org)

Today we’re going to look at two specific sequences and see what we can discover.

Generalized “Tetris”
Some of you may have played the game Tetris before. In this game, the following square clusters are used:

\[
\begin{array}{cccc}
\includegraphics{tetris-cluster1} & \includegraphics{tetris-cluster2} & \includegraphics{tetris-cluster3} & \includegraphics{tetris-cluster4} \\
\includegraphics{tetris-cluster5} & \includegraphics{tetris-cluster6} & \includegraphics{tetris-cluster7} & \includegraphics{tetris-cluster8}
\end{array}
\]

There are two pairs here that are the same up to rotation.
How many pieces are there with 4 squares, up to rotation and reflection?

How about 1 square? 2 squares? 3 squares? Draw them here:
Now count how many pieces there are for 5 squares. Can you find the number for 6?

Let’s enter the sequence we found into OEIS!

**Graham sequences**

Let’s look at a finite sequence. This sequence is called $a_n(6)$ and it satisfies the following conditions:

A. $a_n(6)$ is a sequence of positive integers

B. $a_n(6)$ is a finite sequence

C. $a_n(6)$ starts with 6

D. $a_n(6)$ is increasing

E. The product of the terms of the sequence $a_n(6)$ is a perfect square

Understanding the conditions:

1. How many different possibilities are there for which one of these conditions is true? For example, A,C, and D could be true, or all A-E conditions could be true, or none of them could be true, etc.

2. Find five sequences so that each one satisfies exactly one of the properties. (For example, find a sequence that satisfies A, but not B-E. And then find a sequence that satisfies B but not A,C,D, or E.)
3. Find four sequences that satisfy all five conditions.

The conditions $A - E$ can be generalized for sequences starting with any positive integer $m$. In this case, we will call the properties $A(m), \ldots, E(m)$. Using these properties, Ron Graham defined the following sequence:

$$g_m = \min(\max(\{a_n(m) \text{ satisfying } A(m), \ldots, E(m)\})).$$

4. Find the maximum of each of your sequences from question 3.

5. Find the minimum of all possible maximum values.

6. How can you know that you found the minimum of all possible maximum values?

7. Compute the first ten terms of the Graham sequence.

8. Does 20 appear in the Graham sequence somewhere? If so, where? If not, why not?

9. Does 7 appear in the Graham sequence somewhere? If so, where? If not, why not?

10. How about 120?

11. Can you decide exactly which numbers appear in the Graham sequence?