1. Use the following graph to prove: $1+3+5+\cdots+(2n-1) = n^2$

2. Use the following graph to prove: $\frac{1}{3} = \frac{1+3}{5+7} = \frac{1+3+5}{7+9+11} = \frac{1+3+5+7}{9+11+13+15}$

3. Arithmetic mean and geometric mean

Given two numbers $a$ and $b$, the geometric mean is defined as: $\sqrt{ab}$; the arithmetic mean is defined as: $\frac{a+b}{2}$. Use the following graph to prove $\sqrt{ab} \leq \frac{a+b}{2}$.
4. Left is a $8 \times 8$ chess board with two corner cut off, prove that it is impossible to cut this board into $1 \times 2$ pieces as shown on the right.

What if paint the board as below?

5. Prove that if the two pieces cut off from the chess board have opposite colors, it is always possible to cut the board into $1 \times 2$ pieces.
6. In mathematics, the *Fibonacci numbers* are the numbers in the following integer sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

By definition, the first two numbers in the Fibonacci sequence are 1 and 1, and each subsequent number is the sum of the previous two.

In mathematical terms, the sequence $F_n$ of Fibonacci numbers is defined by the recurrence relation: $F_n = F_{n-1} + F_{n-2}$, with $F_0 = F_1 = 1$.

Use the following graph to prove $F_0^2 + F_1^2 + \cdots + F_n^2 = F_n F_{n+1}$.