Circle Games

1 "Boing-Flip"

This game is played by passing a ball between students arranged in a circle, as in the diagram:

- Before the game starts, everyone in the circle silently chooses a direction: left or right.
- To start the game, the first student with the ball passes it to any other student.
- The student who catches the ball then passes it to a student immediately left or right of the student who passed it, matching the direction he/she chose.
- Every time a student passes a ball, he/she changes the direction for when they receive the ball again: to the left if he/she originally chose right, and to the right if he/she originally chose left.

Questions:

1. Will every student in the circle eventually get a turn to catch and pass the ball?
2 Fair Split

A pair of pirates have acquired a necklace of six black and six white pearls. (Shade in any six of the pearls in the diagram, which need not be all together.) They want to split the necklace into as few pieces as possible so that when they divvy up the pieces of the necklace, they have three white and three black pearls each.

Questions:

1. What is the minimal number of cuts they have to make?

2. Where should the cuts be placed?

3. Do the answers to (a) and (b) change if the necklace has 10 pearls, still with equal amounts black and white? 14 pearls?
3 Blackout!

The country of Flicker has 8 towns arranged in a circle, and the power has gone out in every one of them. When engineers restore or cut power in one town, it affects the power of the two neighboring towns in an odd way: if power in the neighboring town was off, now it is on again, and if the power in the neighboring town was on, now it is off again.

Questions:

1. Number the towns "1" through "8" consecutively. Determine a sequence of towns in which the power should be turned on or turned off in order to restore power to all towns simultaneously.

2. Try to determine a shorter sequence than you found in (a). What do you think is the shortest possible sequence?

3. How might the engineers solve the problem differently if Flicker added a 9th town to the circle? A 10th town? An 11th town?

4. Try to come up with your own variation to this game, and investigate whether it can be solved. If so, how, and in how many moves?

Epilogue: This is a well known type of puzzle. It is similar to the way Stomp is played, but on a circular board. Have you seen Blackout!, Stomp, or similar puzzles anywhere else? (Conway’s Game of Life, Tiger Toy’s “Lights Out”, etc.)
4 Passing Candy

Four sugar-high children just returned from Trick-O-Treating with candy hordes of various sizes, and sit down in a circle. A concerned parent proposes that if each of the children gives half of his/her pile to the child on the left, then any child with an odd number of candies will be given an extra candy to make the pile even. If this process is carried out indefinitely, what happens to the size of each pile? Do the piles even out, and if so, to what amount of candy?

Questions:

1. Randomly choose four positive even integers to represent each child’s pile of candy. Carry out the parent’s idea: divide each pile in half, and add that number to the next pile in the clockwise direction. Then add one to each pile with an odd amount of candy.

2. Without repeating the process, can you make any statements about what will or will not happen to the size of the piles no matter how many times the process is repeated?

3. Repeat the process several times. Then try it with entirely different starting numbers.

4. Do the piles tend to a certain size? How does what happens relate to the original sizes of the piles?

5. Replay the game with 3, 5, and 6 piles. How does this affect the final result, if there is one?