A De Bruijn sequence card trick

De Bruijn sequences

A De Bruijn sequence is a binary cyclic sequence (a list of zeros and ones) such that any possible subsequence of length \( n \) appears exactly once.

Example: What does a De Bruijn sequence with \( n = 2 \) look like? How long does the sequence need to be?

(1) Find a De Bruijn sequence with \( n = 3 \).

(2) How long does a De Bruijn sequence need to be for an arbitrary \( n \)?
The Pigeon Suit

Requires two players. While player A is outside of the room, the audience chooses 5 cards. Player B looks at the cards, and places them face down in any order he or she chooses on the table. Player A returns, turns over the first four cards, and then stuns the audience by knowing the last card!

The goal is to figure out a way to repeat this trick with 52 cards and a five card hand. Consider smaller problems first. \((N,H)\) where \(N\) is the number of cards in the deck, and \(H\) is the number of cards chosen. For each game, try various numbers and distributions of suits.

a) Find a way to do a (3,2) game.
b) Find a way to do a (4,3), (5,3), and (6,3) game.
c) Find a way to do a (7,3) game.
d) Find a way to do a (8,3) game.
e) Repeat the original (52, 5) game.
f) Add one of the jokers to the deck, and find a way to do a (53, 5) game.
g) In general, what is the largest deck \(N\) with which it is possible to play a given hand size \(H\)?