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High School Teacher's Circle
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The University of Arizona
Nested Polyhedra Project

I developed this project and had it published in the May 1994 issue of *The Mathematics Teacher*. It has evolved somewhat since then. Ginny Bohme made up the student worksheets. The project is a blend of geometry skills. The idea is to calculate the dimensions for four of the five Platonic Solids and to build them so that they fit inside each other.

Materials:
- Rulers
- Calculators
- Compasses and protractors
- Glue Sticks
- Poster board
- Scissors
- Icosahedral dice (one for each student)

Procedure:
If your icosahedral dice are all the same size, then you can complete the Teachers Worksheet (included in this packet) and give them the initial edge length of the tetrahedron. If you have a variety of different size dice, then they will need to complete the worksheet first.

For each shape, hold one up to the class and ask them to sketch a net. Have a few students put their nets on the board for discussion.

We start with the tetrahedron because it is the easiest. Don’t let students glue the face with two tabs on it. This is the door. Have them glue only the face with a single tab.

A tetrahedron edge represents a diagonal of the square face of the cube. Therefore to calculate the cube edge length, take the tetrahedron edge length divide by radical 2, then add 2 mm (because of the thickness of the poster board).

The octahedron fits inside of the tetrahedron. Each edge length of the octahedron should be ½ the tetrahedron edge length minus 2 mm (because of the thickness of the poster board).

Each cube edge is a diagonal of each pentagonal face of the dodecahedron. Using trigonometry, calculate the pentagon edge length by multiplying the cube’s edge by 0.618 (1/Φ) then add 2 mm.

Have students show the calculations and draw the net for each of the polyhedra (ws included). Since this project begins with an icosahedral die, its edge length doesn’t need to be calculated. However, the relationship between the octahedron and the icosahedron is quite interesting (see the Platonic Nets page for a diagram). The vertices of the icosahedron cut the edges of the octahedron in the golden ratio, Φ. The law of cosines can then be used to find the edge length of the octahedron.

Students could calculate the surface areas and volumes of each polyhedron as a final aspect of the project (ws included).
Arizona State Proficiency Level Standards addressed:

**Strand 3: Patterns, Algebra, and Functions**  
**Concept 3: Algebraic Representations**  
PO 1. Create and explain the need for equivalent forms of an equation or expression.

**Strand 4: Geometry and Measurement**  
**Concept 1: Geometric Properties**  
PO 2. Visualize solids and surfaces in 3-dimensional space when given 2-dimensional representations. (The two-dimensional representations should include nets.)  
PO 6. Solve problems using angle and side length relationships and attributes of polygons.  
PO 10. Solve problems using right triangles, including special triangles.  
PO 11. Solve problems using the sine, cosine, and tangent ratios of the acute angles of a right triangle.

**Strand 4: Geometry and Measurement**  
**Concept 4: Measurement**  
PO 5. Calculate the surface area and volume of 3-dimensional figures and solve for missing measures.

Reference:  
http://www.ade.state.az.us/standards/math/Articulated08/
Teachers Work Sheet

Use this ws when you start with an icosahedral die and calculate the edge length of the tetrahedron for the student projects.

1. Initial measurement.

Measure the edge length of your icosahedral die to the nearest mm. 

\[ i = \ \boxed{\_\_\_\_\_\_\_\_\_\_\text{cm}} \]

2. Edge length of the octahedron (o).

Add 1 mm to account for the cardboard folds.

\[ + \ \boxed{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\text{cm}} \]

\[ \text{total} \ \boxed{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\text{cm}} \]

Multiply your total by 1.85. Round your answer to a millimeter. Don’t round down unless you are very close to that value. (The 1.85 factor will be explained later.)

\[ \text{Octahedron edge length,} \quad o = \boxed{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\text{cm}} \]

3. Edge length of the tetrahedron(t).

Add 1 mm to the octahedron edge length.

\[ + \ \boxed{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\text{cm}} \]

\[ \text{total} \ \boxed{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\text{cm}} \]

Multiply your total by 2.

\[ \text{Tetrahedron edge length,} \quad t = \boxed{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\text{cm}} \]

This is the edge length your students will use to start their projects.
Nested Polyhedra Calculations

Use this sheet to show the calculations for each of your polyhedra edge lengths and to sketch the net for each. This sheet should be submitted with your completed nested set.

Tetrahedron edge \( (x) = \) _____ cm

Cube edge \( (c) = \frac{x}{\sqrt{2}} = \) ____ cm

Octahedron edge \( (x) = \) _____ cm

Measure of one angle in a regular pentagon:
\[
\frac{(n-2) \times 180}{n}
\]

Calculation of length of side of the pentagon: **SHOW THE WORK!**

Calculation of the radius of the pentagon: **SHOW THE WORK!**
Calculations of Surface Areas and Volumes
For the Nested Set of Regular Polyhedra

1. Tetrahedron:
   - Surface Area - Type of face_________ Number of faces_________
   - Area of one face (Show the calculation. Label what each value represents. Label the units!)
   - Total Surface Area - (Show how. Label the units.)
   - Volume - What formula are you using? ______________
     Explain what each variable in the formula represents. Show the Calculations.
     Label the units.

2. Cube
   - Surface Area - Type of face_________ Number of faces_________
   - Area of one face (Show the calculation. Label what each value represents. Label the units!)
   - Total Surface Area - (Show how. Label the units.)
   - Volume - What formula are you using? ______________
     Explain what each variable in the formula represents. Show the Calculations.
     Label the units.

3. Octahedron
   - Surface Area - Type of face_________ Number of faces_________
   - Area of one face (Show the calculation. Label what each value represents. Label the units!)
   - Total Surface Area - (Show how. Label the units.)
   - Volume - What formula are you using? ______________
     Explain what each variable in the formula represents. Show the Calculations.
     Label the units.
Nets of Platonic Solids

Tetrahedron net.

Octahedron net with an extra face for a door flap.

Icosahedron face inside Octahedron face.

Icosahedron face inside Octahedron face.