RESEARCH STATEMENT **Joseph Stover**

Introduction. The motivation for my graduate research is the problem of invasive species. The term usage is somewhat disputed, but is understood here to refer to recently introduced non-indigenous species. Problems posed by this issue can be devastation of crops, drastic changes in landscape, and the extinction of native species due to habitat loss.

The introduction of grasses from Africa to Southern Arizona for the purpose of grazing cattle is the test case here. Native grasses tend to grow in smaller patches leaving plenty of open area, but this can not sustain ranching in the desired manner leading to the introduction of foreign species such as *Eragrostis Lehmanniana* and *Pennisetum ciliare*. One result is that the ground cover is much more dense leaving behind more fuel for fires in the dry season[1]. Habitats thus created are preferred less by native species of insects, birds, and mammals[5, 1, 2]. These species have spread far beyond what was previously thought to be their limits[11, 6]. My dissertation is centered around the creation of models for the purpose of understanding the spatial spread of an introduced plant species and the development of a general theory for a larger class of models.

The traditional approach to this problem is to use a multi-type contact process and simulate it to equilibrium and then collect data on the equilibrium state. The first problem with this approach is that it is not easy to say exactly when equilibrium is reached. Coupling from the past using the Propp and Wilson algorithm solves this problem[9]. Although if the process is not monotone, another problem arises. This can take a massive number of simulations forcing each possible initial configuration to be tested. A major goal of my dissertation was to create a modified version of the multi-type contact process which remains monotone in biologically meaningful situations. Knowing exactly when equilibrium is reached allows the study of the transient behavior, the path to invasion, and has resulted in a general theory of monotonicity for a larger class of models.

Background

Contact Process. The basic building block for any interacting particle system invasion model is a continuous time, pure jump Markov process called the contact process. It is characterized by a state space, X, composed of configurations of two particle types on a discrete two-dimensional grid, $X = \{0,1\}^{\mathbb{Z}^2}$. A particle of type 0 represents an empty site, which will be available to colonization by a particle of type 1 which should be understood as an adult reproductive plant. There are two types of transitions allowed at each site. A 1 is replaced by a 0 at the constant rate of 1. This means that each plant waits for an exponential clock to go off, with mean time 1, and then dies, leaving behind an empty site. An empty site may become occupied only if it has an occupied neighbor. Each plant neighboring an empty site has an invasive interaction at rate λ. This amounts to a 0 being replaced by a 1 at the exponential rate $\lambda\times$(the number of occupied neighbors)[7].

This is a monotone process which means that given any two initial configurations, η and ξ, with the restriction that wherever η has a 1, ξ also has a 1, but ξ is allowed to have other occupied sites as well, the future states can be constructed on a common probability space in a way which preserves this fact, $\{x : \eta_t(x) = 1\} \subset \{x : \xi_t(x) = 1\}$ for all time[8].

Stationary Sampling. What has been studied before is the spatial distribution of plants at equilibrium. The Propp and Wilson algorithm uses a technique known as *coupling from the past* or CFTP, and each simulation returns an exact sample from the stationary distribution[9]. In many cases, this algorithm is computational unfeasible because a simulation from every initial state is needed. Monotonicity, however, only requires the extremal states where the sites are either all occupied or all vacant as input. Once this algorthim is implemented a detailed picture of the path to invasion is produced.

The approach to invasion by incorporating an invasive species, 2, into the contact process gives us the multi-type contact process whose state space is $\{0,1,2\}^{\mathbb{Z}^d}$[4]. Each species individually acts as a standard contact process. The problem arises that this system is no longer monotone making the study of the stationary distribution unfeasible. The simplest way to allow this process to remain monotone is to allow the invasive species to view a site occupied by the native as empty. This induces an undesired biological condition warranting the need for a new approach.

Current Work

The New Model. In order to retain monotonicity it is necessary to introduce what I call *virtual* particles. The new state space is $\{1-, 1+, 2-, 2+\}^{\mathbb{Z}^d}$. the 'minus' states are equated to the 0 state of the original contact process and represent empty sites only available to a single species. The 'plus' slates represent the individual adults of each species. If only species 1 or 2 particles are present, meaning only the $+$ and $-$ varieties of a single number are present, the dynamics are identical to a contact process. The difference is that the minus states communicate by a two way contact interaction, $i-$'s and $i+$'s influence $j-$ to become $i-$. A death of any plant is allowed to leave behind the virtual state of any species. This model enforces a time lag between occupations by different plants. Under certain biologically meaningful parameter restrictions, I have proved that the model is monotone enabling the use of CFTP to sample from the stationary distribution and observe the transient behavior. What the information about the transient path will tell us is explicitly what the invasion looks like. When the species exhibit a degree of clumping together, statistics on this can be calculated. Also the requirements for coexistence can be studied, and an underlying cyclic behavior is often present.

Mean Field Analysis. Even though the model works in spatially heterogeneous environments, we can study this process using a mean field analysis. By taking appropriate limits on the grid size, we can show that the fraction of occupied sites for each type is given by the solution to a differential equation. The qualitative behavior of the mean field model still closely follows that of the stochastic system. This is used to look for initial conditions and parameter values that may be of biological interest.

Coupling. The efficiency in computation for these models is based upon having an effective coupling. Due to the exponential distribution attached to the times between transitions, Poisson point processes can be used to build process realizations. A Poisson point process is a Poisson number of points uniformly distributed on $(0, t) \times (0, \lambda)$, and the distance between them along the first coordinate, representing time, is exponentially distributed with parameter λ. Once a point process is created, the process can be simulated from all initial configurations. This gives a comprehensive view of the transient paths.

Monotone Process Theory. Many models have been studied which are essentially some variation of a contact process. Examples include the lattice Lotka-Volterra model[12] and bacterial colicin models[4]. The common factor in these types of models is that each interaction between two particle types has a unique outcome. For an example: in the virtual particle model presented above, when a $2-$ is affected by a neighboring $2+$, the possible result is that the $2-$ becomes a $2+$; a $2+$ will never have any other influence on a $2-$. This led me to introduce the *interaction map*. This map defines what interactions occur between particles. Then what followed was to show that monotonicity requires a restriction on what interactions are allowed and the rates at which they occur. This theory is applicable to any model with the property that interactions are unique in the sense described above.

What this has done is created a framework for studying phenomena using interaction based particle models. One can create a set of possible interaction maps and test to see which allow monotonicity. Once a desirable model is established, mean field analysis is performed to capture a broad sense of the behavior, then CFTP is used to sample from the stationary distribution and analyze the transient behavior. Statistics of scientific interest can then be looked at.

Future Work

Further study of the virtual particle model will incorporate spatially and temporally heterogeneous parameters to account for seasonality and variations in geography. The inclusion of a greater number of species will also lead to a more realistic simulation as will comparing simulation data to that observed by field ecologists. In particular the Santa Rita Experimental Range has decades worth of species distribution and density information including the introduced species mentioned above. Seeing the parallels between simulation and reality is the originating objective for modeling.

Within this general framework of monotonicity, there are endless avenues of exploration. How previously studied models fit in or whether they can be modified in a similar fashion as I did to the multi-type contact process is something worth exploring. The methods I have used for plant invasion models can be applied to similar problems such as infectious diseases[10] or predator mediated coexistence[4]. Another popular research area is modeling tumor growth in cancer cells. Interacting particle system type models have been used to study tumor induced blood vessel formation[3]. Particle models are relatively new on the scene and have a lot of room to grow and contribute to the complex facets of modern science.

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