Assignment 3
Probability

Math 363

October, 2009

1. A lake contains 26 lake trout. Twelve are captured, tagged, and released. A certain time later, 5 of the 26 are captured. What is the probability that exactly one of them is tagged? State clearly what assumptions you are using.

2. 5% of the men and 0.25% of the women are color blind.

   (a) Assuming that the population is made up 1/2 men and 1/2 women, find the fraction of the population that is color blind.

   (b) If a person chosen at random is color blind, find the probability that the person is male.

   (c) If two randomly chosen individuals are color blind, find the probability that both are male.

3. Let X be a discrete random variable with probability mass function

x	1	2	3	4	5
$f_X(x)$	c	$2c$	$3c$	$2c$	c

   (a) Find c.

   (b) Find $P\{X \leq 3\}$

   (c) Draw the cumulative distribution for X.

   (d) Find EX, $EX(X-1)$ and $\text{Var}(X)$.

4. For the random variable X.

   (a) Show that

$$F_X(x) = \frac{1}{1+\exp(-x)}, \quad x \in \mathbb{R}.$$

   is a valid cumulative probability distribution function.

   (b) Find the median of X.

   (c) Find the probability density for this distribution function.

   (d) Find the probabilities $P\{-1 < X \leq 0\}, P\{0 < X \leq 1\}$, and $P\{-1 < X \leq 1\}$.

5. The **Pareto random variable** with parameters $\alpha > 0$ and $\beta > 0$ has probability density function

$$f_X(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \quad \alpha < x < \infty.$$

(a) Verify that f_X is a density function.

(b) Find $P\{\alpha < X \le 2\alpha\}$.

(c) Find the mean and variance of X. What restriction do you have on β in computing the variance?

6. In this problem, we shall use R to calculate and simulate with random variables.

(a) For X a negative binomial with $n = 3$ and $p = 3/4$, find $P\{X = x\}$ for $x = 0, 1, \cdots, 10$.

(b) For X a gamma random variable with $\alpha = 4$ and $\beta = 2$, find $P\{X \le x\}$ for $x = 0, 1, 2, 3$. Indicate these values on a plot of the cumulative distribution function.

(c) For Z a standard normal, find values for z so that $P\{Z \le z\} = 0.05, 0.25, 0.50, 0.75, 0.95$ Indicate these values on a plot of the cumulative distribution function.

(d) Simulate 1000 independent beta random variables with $\alpha = 2$ and $\beta = 4$. Find the mean and variance of this sample and compare it to the actual values.

(e) Compare the probability mass functions of a $Bin(200, 1/100)$, and $Bin(2000, 1/1000)$ and $Pois(2)$ random variable by displaying them in a `data.frame`.