Direction Fields

June 8, 2016

To this point, we have discussed both explicit and implicit solutions. These are certainly the goals for the solutions of a differential equations. In the next two sections, we will introduce to additional methods, one graphical and one a numerical approximation, that will add to or ability to understand an initial value problem for a first order differential equation. For the initial value problem

$$y' = f(x, y), \quad y(x_0) = y_0, \tag{1}$$

To introduce the graphic approach, note that at every point of the xy-plane, the differential equation (1), provides a slope $f(x, y)$. For example, for

$$f(x, y) = .x^2 - y$$

we form the following table,

		x				
		-2	-1	0	1	2
	2	2	-1	-2	-1	2
	1	3	0	-1	0	3
y	0	4	1	0	1	4
	-1	5	2	1	2	5
	-2	6	3	2	3	6

For each entry on this table, we place at the indicated point (x, y) a short line with slope $f(x, y)$. Such a graphical representation of a first order ordinary dimensional equation is called a **direction field** or a **slope field**. This representation is useful because we can explore the qualitative behavior of the solutions to the initial value problem without solving the differential equation.

Notice that:

- The function $f(x, y)$ is symmetric in x and skew symmetric in y.
- The direction field will have horizontal lines at the points $(-1, 1)$, $(1, 1)$, and $(0, 0)$.
- Goes from negative to positive at $y = 2$.

Exercise 1. *Check these properties for the slope field represented in Figure 1.*

Figure 1 shows a representation for the slope field along with solution with two initial conditions. These solutions are commonly called **integral curves**.

Exercise 2. *Build the table above for the slope field $f(x, y) = x + y$ and draw integral curves through several points.*

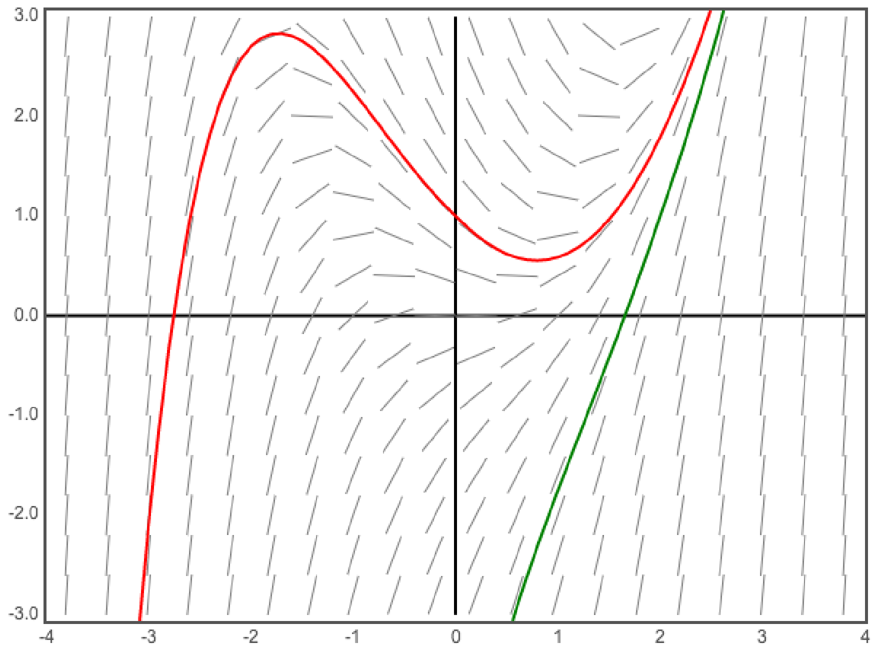

Figure 1: Slope field for the differential equation $y' = f(x, y)$. The curves show the solution this equation with $y(0(= 1$ (in red) and $y(2) = 1$ (in green), (from `http://www.bluffton.edu/ nesterd/java/slopefields.html`