

2. Let X_1 and X_2 be two independent measurements of some unknown value μ . X_1 has higher variance than X_2 .

(a) Should your estimate for μ be closer to X_1 , to X_2 or be the simple average $(X_1 + X_2)/2$? Explain your choice.

(b) To test this, let X_1 and X_2 be normal random variables with mean μ and respective variances $\sigma_1^2 = 1/2$ and $\sigma_2^2 = 1/20$. Thus, the densities are

$$f_{X_1}(x_1) = \frac{1}{\sqrt{\pi}} \exp -(x_1 - \mu)^2, \quad f_{X_2}(x_2) = \frac{\sqrt{10}}{\sqrt{\pi}} \exp -10(x_2 - \mu)^2.$$

Give the likelihood for the pair X_1, X_2 .

(c) Find the log of the likelihood and use this to find the maximum likelihood estimator $\hat{\mu}$ for μ .

(d) Let $x_1 = 3.11$ and $x_2 = 3.22$. Find the estimate $\hat{\mu}$.

(e) Does this answer support your claim in part (a)?

3. To test the degree of Africanization in a bee hive, a sample of 278 eggs were randomly chosen from a comb. 111 had an Africanized patriline and 167 had a European patriline.

(a) Give the sample proportion of Africanized honey bee eggs.

(b) Give a 98% confidence interval for the population proportion of Africanized honey bees in the hive.

(c) What would happen to the width of the confidence interval if we had sampled 158 eggs? Explain your answer.

4. The Food and Nutrition Board of the National Academy of Sciences states that the recommended daily allowance of iron for adult females under the age of 51 is 18 mg. You suspect that women do not take enough iron.
- (a) State an appropriate null and alternative hypothesis for this situation. State what population parameter you are using.
- (b) For a 24-hour period, 41 randomly selected women had their iron intakes monitored. The sample mean for these women was $\bar{x} = 16.8$. The sample standard deviation was $s = 3.08$. Compute the t statistic associated to the hypothesis test in part (a).
- (c) Make a drawing of the t density curve and shade in the area corresponding to the p -value.
- (d) Is the test significant at the 5% level? At the 1% level?

5. You want to use hypothesis testing to refute the statement that men and women are the same age at the time of their marriage, stating that the traditional fact that husbands are older to their wife's still holds.

(a) Write an appropriate hypothesis test for this situation and state the testing procedure appropriate to this circumstance.

(b) Below is a table of the ages in years of 6 randomly chosen couples at the time of their marriage.

Husband's age	43	57	30	19	33	39
Wife's age	37	51	32	20	31	38

Compute the necessary summary statistics for the test in part (a).

(c) Perform the t -test and make a statement about the p -value based on the tabular entries.

(d) How would the p -value change for a two-sided t -test?