Mathematics 363 - Exam II

April 27, 2009

1. For a fair die, the number of dots on the top side has mean $\mu = 7/2$ and variance $\sigma^2 = 35/12$. Let $\bar{X}$ be the sample mean for 100 rolls of the die.

(a) Find the mean and variance of $\bar{X}$.

(b) You suspect that the dice favors high numbers and will announce your suspicion if the sample mean is above 3.75. Use the central limit theorem to estimate $P\{\bar{X} > 3.75\}$.

(c) If the number of rolls increases to 400 and if the die is indeed fair, then will $P\{\bar{X} > 3.75\}$ increase, decrease, or stay the same? Explain your answer.
2. Let $X_1$ and $X_2$ be two independent measurements of some unknown value $\mu$. $X_1$ has higher variance than $X_2$.

(a) Should your estimate for $\mu$ be closer to $X_1$, to $X_2$ or be the simple average $(X_1 + X_2)/2$? Explain your choice.

(b) To test this, let $X_1$ and $X_2$ be normal random variables with mean $\mu$ and respective variances $\sigma^2_1 = 1/2$ and $\sigma^2_2 = 1/20$. Thus, the densities are

$$f_{X_1}(x_1) = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{(x_1 - \mu)^2}{2}\right), \quad f_{X_2}(x_2) = \frac{\sqrt{10}}{\sqrt{\pi}} \exp\left(-\frac{10(x_2 - \mu)^2}{20}\right).$$

Give the likelihood for the pair $X_1, X_2$.

(c) Find the log of the likelihood and use this to find the maximum likelihood estimator $\hat{\mu}$ for $\mu$.

(d) Let $x_1 = 3.11$ and $x_2 = 3.22$. Find the estimate $\hat{\mu}$.

(e) Does this answer support your claim in part (a)?
3. To test the degree of Africanization in a bee hive, a sample of 278 eggs were randomly chosen from a comb. 111 had an Africanized patriline and 167 had a European patriline.

(a) Give the sample proportion of Africanized honey bee eggs.

(b) Give a 98% confidence interval for the population proportion of Africanized honey bees in the hive.

(c) What would happen to the width of the confidence interval if we had sampled 158 eggs? Explain your answer.
4. The Food and Nutrition Board of the National Academy of Sciences states that the recommended daily allowance of iron for adult females under the age of 51 is 18 mg. You suspect that women do not take enough iron.

(a) State an appropriate null and alternative hypothesis for this situation. State what population parameter you are using.

(b) For a 24-hour period, 41 randomly selected women had their iron intakes monitored. The sample mean for these women was \( \bar{x} = 16.8 \). The sample standard deviation was \( s = 3.08 \). Compute the \( t \) statistic associated to the hypothesis test in part (a).

(c) Make a drawing of the \( t \) density curve and shade in the area corresponding to the \( p \)-value.

(d) Is the test significant at the 5% level? At the 1% level?
5. You want to use hypothesis testing to refute the statement that men and women are the same age at
the time of their marriage, stating that the traditional fact that husbands are older to their wife's still
holds.

(a) Write an appropriate hypothesis test for this situation and state the testing procedure appropriate
to this circumstance.

(b) Below is a table of the ages in years of 6 randomly chosen couples at the time of their marriage.

<table>
<thead>
<tr>
<th>Husband's age</th>
<th>43</th>
<th>57</th>
<th>30</th>
<th>19</th>
<th>33</th>
<th>39</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wife's age</td>
<td>37</td>
<td>51</td>
<td>32</td>
<td>20</td>
<td>31</td>
<td>38</td>
</tr>
</tbody>
</table>

Compute the necessary summary statistics for the test in part (a).

(c) Perform the \( t \)-test and make a statement about the \( p \)-value based on the tabular entries.

(d) How would the \( p \)-value change for a two-sided \( t \)-test?