Mathematics 363 - Final Exam

Brief Answers

May 14, 2009

1. (a) mean deceleration - 4.3, variance of deceleration - 10.87333, the covariance of time and deceleration - 42.33333
(b) deceleration = 0.490 + 0.254 time
(c) -0.33
(d) 0.254

2. (a) \( P(S^c) = 0.999 \)
(b) \( P(T|S) = 0.99 \)
(c) \( P(T) = 0.99 \times 0.001 + 0.05 \times 0.999 = 0.05094 \)
(d) \( P(S|T) = P(T|S)P(S)/P(T) = 0.99 \times 0.001/0.05094 = 0.01943 \)
(e) Reducing false positives by 20% to 0.04. Since most people do not have the flu, improving the test for those without the disease will have a bigger impact.

3. (a) The mean is of \( \bar{F} \) 440 Hertz and standard deviation of \( \bar{F} \) is \( 2/\sqrt{10} \).
(b) For \( g(F) = v/\bar{F}, \ g'(F) = -v/F^2 \). So, the mean is \( 343/440 = 0.780 \) m.
\( \sigma_\lambda = (v/F^2) \times \sigma_\lambda/\sqrt{10} = (343/440^2) \times 2/\sqrt{10} = 0.00112 \). Square this to obtain the variance.
(c) Increase - the denominator in the expression above decreases.

4. (a) area is a trapezoid - left is \( x = 0 \), right is \( x = 1/2 \), bottom is \( y = 1 \) and top is \( y = 1/2 + x \)
(b) 3/8
(c) \( EX = 1/2 + \theta/12 \)
(d) \( \hat{\theta} = 12\bar{X} - 6 \)
(e) 1.2

5. (a) 0.885. The z score is -1.2. Shade the area to the right of \( z = -1.2 \) below the standard normal density curve.
(b) The 40% quantile is \( z = -0.2533 \). So diameters greater than 12.33.
(c) The mean of \( \bar{X} \) is 14. The variance is 25/50 = 1/2.

6. (a) For \( p \) the fraction of of teen girls in Arizona became mothers in a year,
\[ H_0 : p \geq 0.062 \quad H_0 : p < 0.062. \]
(b) \( \hat{p} \) has mean \( p_0 = 0.062 \) and standard deviation \( \sqrt{p_0(1 - p_0)/1600} = 0.00607 \)

(c) \( z = -0.2623 \). The \( p \)-value is 0.396, too high to reject.

7. (a) For \( \mu_w \) and \( \mu_r \) the mean weight of wren and robin eggs, respectively,

\[
H_0 : \mu_r = \mu_w \quad H_0 : \mu_r \neq \mu_w
\]

(b) \( t = (21.063 - 22.575)/\sqrt{0.776^2/17 + 0.685^2/16} = -5.942 \). This is strong evidence against \( H_0 \).

(c) No. For the 98\% confidence interval, \( t^* = 2.467 \)

8. (a) Let \( p_t \) be the fraction of the population having blood type \( t = A, B, AB, O \)

\[
H_0 : p_A = 0.42, p_B = 0.10, p_{AB} = 0.04, p_O = 0.44, \quad H_1 : \text{at least one of the } p_t \text{ differs from these values.}
\]

(b) \[
\begin{array}{cccc}
A & B & AB & O \\
42 & 10 & 4 & 42 \\
\end{array}
\]

(c) The chi-square statistics is 7.7219. The \( p \)-value is just above 0.05.