Mathematics 363 - Final Exam

Brief Answers

May 14, 2009

1. (a) mean deceleration - 4.3, variance of deceleration - 10.87333, the covariance of time and deceleration - 42.33333
   (b) deceleration = 0.490 + 0.254 time
   (c) -0.33
   (d) 0.254

2. (a) $P(S^c) = 0.999$
   (b) $P(T|S) = 0.99$
   (c) $P(T) = 0.99 \times 0.001 + 0.05 \times 0.999 = 0.05094$
   (d) $P(S|T) = P(T|S)P(S)/P(T) = 0.99 \times 0.001/0.05094 = 0.01943$
   (e) Reducing false positives by 20% to 0.04. Since most people do not have the flu, improving the test for those without the disease will have a bigger impact.

3. (a) The mean is of $\bar{F}$ 440 Hertz and standard deviation of $\bar{F}$ is $2/\sqrt{10}$.
   (b) For $g(F) = v/\bar{F}$, $g'(F) = -v/F^2$. So, the mean is 343/440 =0.780 m.
   $\sigma_{\hat{\lambda}} = (v/F^2) \times \sigma_\lambda/\sqrt{10} = (343/440^2) \times 2/\sqrt{10} = 0.00112$. Square this to obtain the variance.
   (c) Increase - the denominator in the expression above decreases.

4. (a) area is a trapezoid - left is $x = 0$, right is $x = 1/2$, bottom is $y = 1$ and top is $y = 1/2 + x$
   (b) 3/8
   (c) $EX = 1/2 + \theta/12$
   (d) $\hat{\theta} = 12\bar{X} - 6$
   (e) 1.2

5. (a) 0.885. The z score is -1.2. Shade the area to the right of $z = -1.2$ below the standard normal density curve.
   (b) The 40% quantile is $z = -0.2533$. So diameters greater than 12.33.
   (c) The mean of $\bar{X}$ is 14. The variance is $25/50 = 1/2$.

6. (a) For p the fraction of of teen girls in Arizona became mothers in a year,

$$H_0 : p \geq 0.062 \quad H_0 : p < 0..062.$$

(b) $\hat{p}$ has mean $p_0 = 0.062$ and standard deviation $\sqrt{p_0(1-p_0)/1600} = 0.00607$

(c) $z = -0.2623$. The p-value is 0.396, too high to reject.

7. (a) For μ_w and μ_r the mean weight of wren and robin eggs, respectively,

$$H_0 : \mu_r = \mu_w \quad H_0 : \mu_r \neq \mu_w$$

(b) $t = (21.063 - 22.575)/\sqrt{0.776^2/17 + 0.685^2/16} = -5.942$. This is strong evidence against H_0.

(c) No. For the 98% confidence interval, $t^* = 2.467$

8. (a) Let p_t be the fraction of the population having blood type $t = A, B, AB, O$

$H_0 : p_A = 0.42, p_B = 0.10, p_{AB} = 0.04, p_O = 0.44, \quad H_1 :$ at least one of the p_t differs from these values.

(b)

A	B	AB	O
42	10	4	42

(c) The chi-square statistics is 7.7219. The p-value is just above 0.05.