Mathematics 363 - Final Exam

May 14, 2009

Do any 7.

1. During Pathfinder's approach to Mars, the Martian atmosphere was used to slow down the vehicle. For the 30 seconds of this entry phase, we show time in seconds and the vehicle deceleration in G's, multiples of the acceleration of gravity on Mars.

time	0	10	20	30
deceleration	0.8	2.7	5.3	8.4

The mean of the explanatory variable seconds is 15 seconds and its variance is 500/3 seconds^2.

(a) Compute the mean and variance of deceleration and the covariance of time and deceleration.

(b) Compute the least squares regression line of vehicle deceleration on time.

(c) Give the residual value at 10 seconds.

(d) According to the regression line, how much did the Pathfinder decelerate each second on average during this period.

2. Labs all over the world are frantically trying to develop an instant test for swine flu. Let S be the event that a person in Mexico City has swine flu and let T be the event that a person tests positive for swine flu. Any test that is found will have *false positives*, a positive test for a patient that does not have swine flu, and *false negatives*, a negative test for a patient that does have swine flu. Assume that we have a test for swine flu with a false positive rate

$$P(T|S^c) = 0.05$$

and a false negative rate $\quad P(T^c|S) = 0.01$.
Assume that the the fraction of the Mexico City population that has swine flu, $P(S) = 0.001$. Find

(a) $P(S^c)$

(b) $P(T|S)$

(c) $P(T)$

(d) $P(S|T)$

(e) To improve the value of $P(S|T)$ what is more valuable - reducing false positives by 20% to 0.04 or educing false negatives by 20% to 0.008? Explain your answer.

3. Wavelength λ is determined using the formula

$$\lambda = \frac{v}{f},$$

where v is the phase speed of the wave and f is its frequency. For sound waves in air, this is the speed of sound, 343 m/s in air at room temperature and atmospheric pressure.

(a) A sound pressure meter can measure frequency with a standard deviation $\sigma_f = 2$ Hertz or cycles per second. A standard pitch A at 440 Hertz is measured 10 times, $F_1, \ldots, F_{10}$ and averaged to give $\bar{F}$. Give the mean and standard deviation of $\bar{F}$.

(b) Let $\hat{\lambda} = v/\bar{F}$ be an estimator for the wavelength. Use the delta method to estimate the mean and variance of $\hat{\lambda}$.

(c) If we measure a lower pitch with a frequency of 220 Hertz, does the standard deviation of $\hat{\lambda}$ increase or decrease. Explain your answer.

4. Let X be a continuous random variable with density

$$f_X(x|\theta) = \begin{cases} 1 - \frac{\theta}{2} + \theta x & \text{if } 0 < x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Sketch the density for the choice $\theta = 1$ and indicate on the sketch an area equal to $P\{X \leq \frac{1}{2}\}$.

(b) Compute this probability.

(c) For an arbitrary value of θ, find the expected value of X.

(d) Use this to find $\hat{\theta}$, a method of moment estimator for θ.

(e) Compute $\hat{\theta}$ for a sample mean $\bar{X} = 0.6$.

5. The diameter of 100 year old ponderosa pine trees in a certain habitat is approximately normally distributed, with mean 14 inches and standard deviation 5 inches.

   (a) If the timber company decides to harvest all trees above 8 inches in diameter, what fraction of the trees will they harvest? Show a sketch of this fraction using a normal density curve labeling the horizontal axis.

   (b) The timber company decides to harvest the 60% of the trees with the largest diameters. What is the smallest tree harvested?

   (c) You suspect that a forest is unhealthy and hypothesize that the mean size is less than 8 inches for a given grove. Under the null hypothesis, find the mean and variance of $\bar{X}$ for a random sample of size 50.

6. The Center for Disease Control states that 6.2% of teen girls in Arizona became mothers in 2006. Arizona health officials said in April that they plan to seek more than \$1 million in federal grant money for abstinence-only education in schools.

   (a) Give an appropriate null and alternative hypothesis to try to show that this education reduces teen pregnancy rates. State what your population parameters are.

   (b) Let $\hat{p}$ be the proportion of Arizona teen girls in a random sample of 1600 that became pregnant in the year after receiving abstinence-only education in schools. What is the mean and standard deviation of $\hat{p}$ under the null hypothesis?

   (c) If $\hat{p} = 0.06$, can the proponents of this educational strategy claim that teen pregnancies have been reduced?

7. Cuckoos are knows to lay their eggs in the nests of other (host) birds. The eggs are then adopted and hatched by the host birds. For two bird species, we have the following summary of the data.

species	# observations	mean	standard deviation
wren	17	21.063	0.776
robin	16	22.575	0.685

(a) State an appropriate null and alternative hypothesis to test that the mean size of a cuckoo eggs are different between wrens' and robins' nests. State what population parameters you are using.

(b) Compute the appropriate test statistic and make a statement concerning the strength of evidence against the null hypothesis. (The Welch formula gives 28 degrees of freedom.)

(c) Would 0 be included in a 98% confidence interval of the difference in the two parameters given in part (a)? Explain your answer.

8. The Red Cross recommends that a blood bank maintains 42% blood type A, 10% blood type B, 4% blood type AB, and 44% blood type O. You suspect that the distribution of blood types in Tucson is not the same as the recommendation.

   (a) State an appropriate null and alternative hypothesis for this situation.

   (b) You plan to collect a random sample of the blood of 100 people and in preparation for a chi square test, compute the expected number of observations of each blood type. Give the result of this computation.

A	B	AB	O

   (c) As a result of you sample, you find the following:

A	B	AB	O
31	10	2	57

   Evaluate the hypothesis in part (a) based on these data.

Have a good summer and congratulations to all those graduating.