1. Let $X$ be a discrete random variable with probability mass function
\[
\begin{array}{c|cccccc}
  x & 0 & 1 & 2 & 3 & 4 & 5 \\
  f_X(x) & c & 3c & c & 3c & c & 3c \\
\end{array}
\]

(a) Find $c$.
(b) Find $P\{X < 2\}$ and $P\{X \leq 2\}$.
(c) Draw the cumulative distribution for $X$.
(d) Let $Y = \sin(\pi X/2)$. Give the probability mass function for $Y$.

2. A random variable $X$ with distribution function
\[
F_X(x) = \exp(-1/x^2) \quad x > 0.
\]

is an example of a Fréchet distribution.

(a) Give a graph of $F_X$ and explain using this plot why $F_X$ is a valid cumulative probability distribution function.
(b) Find the values of the 4 quintiles $X$ and show their values on the graph.
(c) Make a table of $x$ and $F_X(x)$ for $x = 1, 2, \ldots, 10$.
(d) Find the probabilities $P\{1 < X \leq 5\}$ and $P\{5 < X\}$.
(e) Find the probability density for this distribution function.
(f) Provide a second sketch of the distribution function along with a sketch of the density function indicating $P\{1 < X \leq 5\}$ on both plots.