Maximum Likelihood Estimation*

Worksheet 17

1. For a parameter $\theta > 0$, we model the accuracy of a dart player by the $Beta(\theta, 1)$ density

$$f_X(x|\theta) = \begin{cases} 0 & \text{if } x < 0, \\ \theta x^{\theta-1} & \text{if } 0 \le x < 1, \\ 0 & \text{if } 1 \le x, \end{cases}$$

for a continuous random variable X, the distance the dart is from the center of the board.

   (a) For n observations, give the likelihood function.

   (b) Find the maximum likelihood estimate for observations $x_1, \ldots, x_n$.

   (c) Give 10000 simulations of maximum likelihood estimates of $\hat{\theta}$ based on the values $\theta = 1/4, 1/2, 3/4$ and $n = 50$. Give the mean and standard deviation of the simulation for both values of $\hat{\theta}$.

   (d) How do the means of your simulations compare to the actual values of θ.

   (e) How do the standard deviations from the simulations compare to the standard deviation estimate given by the Fisher information?

2. Daily rainfall data, in millimeters, is modeled as having a $\Gamma(1/2, \beta)$ distribution. The density is

$$f_X(x|\alpha, \beta) = \begin{cases} 0 & \text{for } x < 0, \\ \frac{\beta^{1/2}}{\sqrt{\pi}} x^{-1/2} e^{-\beta x} & \text{for } x \ge 0. \end{cases}$$

   (a) Find the maximum likelihood estimator for β based on rainfall amounts $x_1, x_2, \ldots, x_n$

   (b) Compute the Fisher information to estimate to variance of this estimator.

   (c) How does it compare to the method of moments estimator?

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