d) If A and B are $n \times n$ matrices and B is invertible, then $\det(B^{-1} A B^{T}) = \det A$

e) $\det A = 0$ if and only if one of the eigenvalues of A is zero

f) Similar matrices have the same eigenvalues.

g) Similar matrices have the same eigenvectors.

h) If v_1 and v_2 are eigenvectors of a matrix A, then they are linearly independent.

(17) Find the values for x for which $\det \begin{bmatrix} 3 & -1 & 1 \\ 1 & x & -2 \\ -1 & 2 & -1 \end{bmatrix} < 0$

~~(18)~~ A 7×7 matrix has three eigenvalues. One eigenspace is 2-dim, and another is 3-dim. Is it possible that the matrix is <u>not</u> diagonalizable? Explain carefully.

(19) Determine with reasons, whether $A \sim B$. If $A \sim B$ give an invertible matrix s.t $P^{-1} A P = B$.

$A = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 2 \\ 3 & 2 \end{bmatrix}$

~~(20)~~ Let $A = \begin{bmatrix} 2 & k \\ 1 & 0 \end{bmatrix}$. Find all values of k for which:

a) A has eigenvalues 3 and -1.

b) A has an eigenvalue with alg. multip. 2

c) A has no real eigenvalues.

~~(21)~~ Let S be the set of all ordered pairs of real numbers (x, y). Define

$(x_1, y_1) + (x_2, y_2) = (3y_1 + 3y_2, -x_1 - x_2)$

$c(x_1, y_1) = (3cy_1, -cx_1)$.

Show that S with these operations in not a vector space; state explicitly which axioms fail to hold.

~~(22)~~ Let W_1 and W_2 be subspaces of $\mathbb{R}^3$. Show by example that $W_1 \cup W_2$ (union) need not to be a subspace of $\mathbb{R}^3$.