Homework 5
Math 456/556

1. Book exercise 4.1.2.

2. Book exercise 4.1.6.

3. Book exercise 4.2.4.

4. For the wave equation with Dirichlet boundary conditions $u(0,t) = 0 = u(L,t)$, we found the temporal frequencies

$$\omega_n = \frac{n\pi c}{L}, \quad n = 1, 2, 3, ...$$

when we separated variables, i.e. the time-dependent parts had the form $\sin(\omega t)$ or $\cos(\omega t)$. The lowest frequency ω_1 is sometimes called the *fundamental* frequency (for sound waves, this is the pitch of the note you hear). The other temporal frequencies are called *harmonics*, which in this case are just multiples of the fundamental

$$\omega_n = n\omega_1, \quad n = 2, 3, 4, ...$$

(By the way, the presence of other harmonics defines the *timbre* or tone of the sound you hear. $n = 2$ is one octave above the fundamental, $n = 3$ is an octave and fifth above, etc.)

A. Now consider the boundary conditions $u_x(0,t) = 0 = u(L,t)$. By solving the appropriate eigenvalue problem, determine the (temporal) frequencies in this case, and determine what *multiples* of the fundamental frequency are present.
B. Write down the solution to the initial value problem if $L = \pi/2$, $c = 1$, $u(x,0) = \cos(3x)$ and $u_t(x,0) = \cos(7x)$.