Homework 6
Math 456/556

1. Book exercise 6.1.6. Hint: Look for a solution of the form $u = u(r)$, so that u solves an ODE.

2. Book exercise 6.2.4. (Hint: split into two problems, one which has homogeneous boundary conditions on the top and bottom, the other on the left and right)

3. A. Book exercise 6.3.4: show $P(r, \theta)$ solves Laplace's equation.

B. What is $\lim_{r \to a} P(r, \theta)$? (Be careful when $\theta = 0$!)

C. Can the maximum principle be applied to P (you might want to look at exercise 6.1.12)? If so, what is the maximum value of P?

4. Suppose u solves Laplace's equation on the unit disk, $u(0, 0) = 0$ and $u(1, \theta) = 3$ on the *upper half circle* $0 \leq \theta \leq \pi$.

A. Show that the minimum value for u on the disk $r \leq 1$ must be less than or equal to -3.

B. Explicitly construct a solution for which this is true.

5. Book exercise 6.4.10.