Math 583A Fall 2011 Problem Set #8

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Due end of day **Wednesday, 12/7**

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1. Let $U = \frac{1}{\sqrt{2\pi}}\mathcal{F}$, i.e., U is a rescaled version of the Fourier transform. For any function[1] f, compute $U^4 f$.

2. Find the Fourier transforms:

   (a) $\mathrm{sinc} * \mathrm{g}$, where $g(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ and where $\mathrm{sinc(x)} = \mathrm{sin(x)/x}$

   (b) $xg(x)$ (g as above)

   (c) sinc^2

   (d) $s_a * \mathrm{sinc}$, where $s_a(x) = \cos(ax)$

   (e) $\frac{1}{1+x^4}$

   (f) $x^2 \,\mathrm{sinc(x)}$

3. Using properties of the Fourier transform, evaluate

   (a) $PV \int_{-\infty}^{\infty} \mathrm{sinc(x)}\ \mathrm{dx}$

   (b) $\int_{-\infty}^{\infty} \mathrm{sinc(x)}^4\ \mathrm{dx}$

[1] For this problem, you can assume the function is Schwartz, so that the Fourier transform can be iterated. However, note as I explained in class, the Fourier transform extends naturally to L^2, and hence (because Schwartz functions are dense in L^2) your result will hold for L^2 functions as well.