1. Consider the system \( X'(t) = AX(t) \) with
\[
\begin{bmatrix}
5 & 0 & 6 \\
3 & 3 & 7 \\
-3 & 0 & -4
\end{bmatrix}
\] (1)

By diagonalizing \( A \), find the solution with initial conditions
\[
X(0) = \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}.
\] (2)

2. Consider the scalar differential equation
\[
x'''(t) + ax''(t) + bx'(t) + cx(t) = 0.
\] (3)

(a) By introducing two new variables, rewrite the equation as a first-order linear system of the form
\[
X'(t) = AX(t), \quad X(t) = \begin{bmatrix}
x(t) \\
y(t) \\
z(t)
\end{bmatrix},
\] (4)
y and \( z \) are the new variables you defined.

(b) Suppose now \( a = b = c = 0 \). Find the general form of solutions to Eq. (3).

(c) When \( a = b = c = 0 \), is the matrix \( A \) you found in (a) diagonalizable?

(d) For \( a = b = c = 0 \), compute the matrix exponential \( e^{tA} \) directly from the power series definition. Using this, find a general solution for Eq. (4). Is it equivalent to your solution in (b)?