HW #4 partial solutions & hints

§7.7 part 2

1 $\int_{\frac{\pi}{8}}^{\frac{\pi}{2}} \frac{\sin(x)\,dx}{\sqrt{\cos(x)}}$

substitute $u = \cos(x)$:

$$\int \frac{\sin(x)\,dx}{\sqrt{\cos(x)}} = -\int \frac{du}{\sqrt{u}}$$

$$= -2\sqrt{u}$$

$$= -2\sqrt{\cos(x)}$$

Converges

2 $\int_0^4 \frac{8+x^6}{x^2}\,dx$

Diverges ∵

$$\lim_{a\to 0} \int_a^4 \frac{8+x^6}{x^2}\,dx$$

$$= \lim_{a\to 0} \left[-\frac{8}{x} + \frac{1}{5}x^5\right]_a^4$$

$$= +\infty$$

3 $\int_0^5 \frac{1}{u^2-25}\,du$

diverges

Partial fractions:

$$\frac{1}{u^2-25} = \frac{A}{u+5} + \frac{B}{u-5}$$

$$= \frac{A(u-5)+B(u+5)}{u^2-25}$$

$A+B=0$

$-5A+5B=1$

$$\Rightarrow \int \frac{1}{u^2-25}\,du = \int \frac{-1/10}{u+5}\,du + \int \frac{1/10}{u-5}\,du$$

$$= -\frac{1}{10}\ln|u+5| + \frac{1}{10}\ln|u-5|$$

4 $\int_0^{\pi/2} \frac{1}{\cos^2(x)}\,dx$

$$= \left[\tan(x)\right]_0^{\pi/2}$$

diverges

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§7.8 see WebAssign