HW 6 partial solutions

§ 8.2 part 2

[1]

$$\text{arc length} = \int_a^b \sqrt{1 + f'(x)}\, dx$$

$$f(x) = \sqrt{x^3}$$

$$f'(x) = \frac{3}{2} x^{\frac{1}{2}}$$

$$\text{arc length} = \int_0^4 \sqrt{1 + \frac{9}{4} x}\, dx$$

Substitute $u = 1 + \frac{9}{4}x$.

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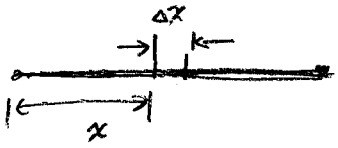
§ 8.4

[1]

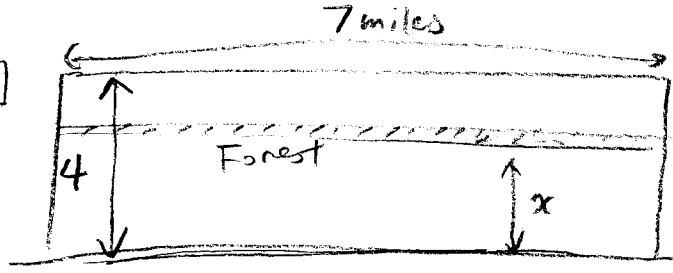


mass of segment $= \Delta x \cdot \delta(x)$

$= \Delta x \cdot (12x + 7)$

$$\text{mass} = \int_0^6 (12x + 7)\, dx$$

[2]



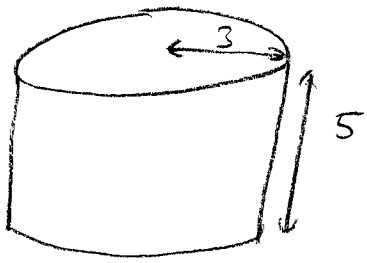
$\frac{\downarrow}{\uparrow} \Delta x$ ← # trees in strip

$= \Delta x \cdot p(x) \cdot 7$

$= \Delta x \cdot \frac{600}{(x+1)^2} \cdot 7$

$$\int_0^4 \frac{4200}{(x+1)^2}\, dx$$

[2]



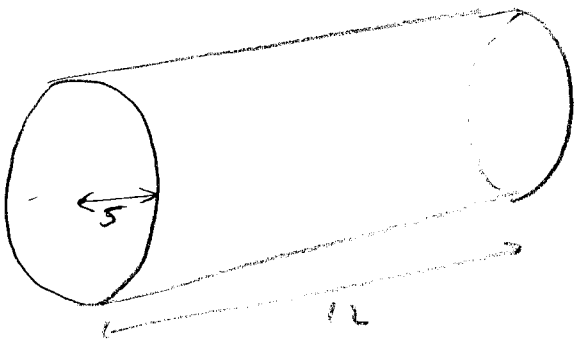
Water density is 62.4 lbs/ft^3

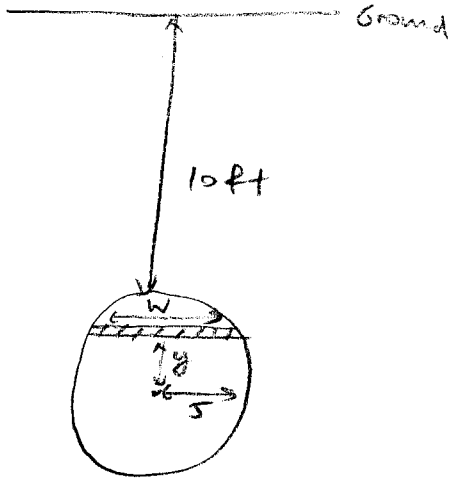
(a) $\boxed{62.4\int_0^5 9\pi(5-y)\,dy}$ (Tank full, pump over top)

(b) $\boxed{62.4\int_0^5 9\pi(7-y)\,dy}$ (Tank full, pump 2 ft above)

(c) $\boxed{62.4\int_0^4 9\pi(5-y)\,dy}$ (Tank filled to 4 ft, pump over top)

[3] (a)

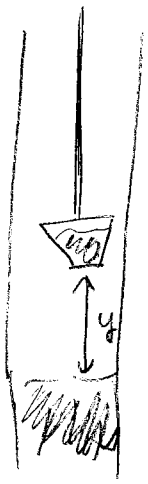




Volume of slice $= w \cdot \Delta y \cdot 12$ ft

$\left(\frac{w}{2}\right)^2 + y^2 = 5^2 \Rightarrow$ volume $= 2\sqrt{25-y^2}\,\Delta y \cdot 12$

$$\boxed{\int_{-5}^{5} 42.24 \cdot \sqrt{25-y^2}\,(15-y)\,dy}$$



Work to raise bucket from height y to $y + \Delta y$:

Force F × distance

$= F \times \Delta y$.

F = weight

$= 50 - \frac{1}{2}y$

So $\boxed{\int_0^{60} (50 - \frac{1}{2}y)\, dy}$ is the work it takes.