HW 7

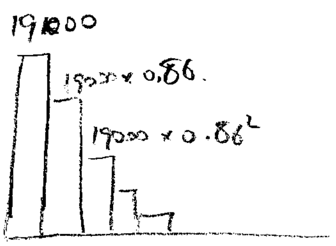
§9.1

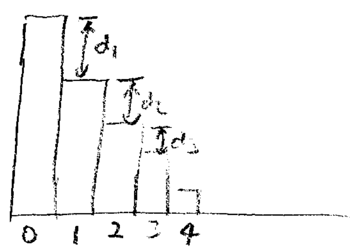
5 (d) $\lim_{n\to\infty} \cos\left(\frac{6}{n}\right) = \cos\left(\lim_{n\to\infty} \frac{6}{n}\right) = \cos(0) = 1.$

(e) $\lim_{n\to\infty} \frac{\sin(n)}{n} = 0 \quad \because \quad |\sin(n)| \leq 1$ for all n.

8 Note: $\cos(\pi n) = (-1)^n$: it just oscillates

10 (a)





$$\boxed{\begin{aligned} d_n &= 19000 \cdot 0.86^{n-1} - 19000 \cdot 0.86^{n} \\ &= 19000 \cdot 0.14 \cdot 0.88^{n-1} \end{aligned}}$$

(Rest is straightforward)

§9.2

$\boxed{2}$ No: The ratio of successive terms in a geometric series is always a constant.

Here, the ratios are not constant!

$$\frac{\text{2nd term}}{\text{1st term}} = \frac{4z}{4} = z$$

but

$$\frac{\text{3rd term}}{\text{2nd term}} = \frac{8z^2}{4z} = 2z.$$

$\boxed{5}$ $3 + 3(0.1) + \cdots + 3(0.1)^{20}$

Expressed in the form

$a + ax + \cdots + ax^{n-1}$

gives $a = 3$ and $x = 0.1$ and $n = 21$ $(n-1=20)$.

$\boxed{7}$ $\sum_{k=4}^{\infty} \left(\frac{1}{4}\right)^k = \left(\frac{1}{4}\right)^4 + \left(\frac{1}{4}\right)^5 + \left(\frac{1}{4}\right)^6 + \cdots$

$= a + ax + ax^2 + \cdots$

$a = \left(\frac{1}{4}\right)^4, \quad x = \frac{1}{4}.$

$$= \frac{a}{1-x} = \frac{(1/4)^4}{3/4} = \frac{4}{3}\left(\frac{1}{4}\right)^4$$

8 (a) $a + ax + ax^2 + \cdots$

Multiply each term by c:

$ca + cax + cax^2 + \cdots$

"New a" $= ca$

x is the same.

(b) Reciprocal of each term:

$a + \frac{a}{x} + \frac{a}{x^2} + \cdots$

"New x" is $\frac{1}{x}$.

a stays the same.

9 (a) Half-life $= 6.5$ hours means

$$e^{a \cdot 6.5} = \frac{1}{2}$$

So $a = \dfrac{-\ln(2)}{6.5}$.

Percentage after 24 hours is $e^{-\frac{24 \cdot \ln(2)}{6.5}} = \boxed{0.08}$.

(b) $Q_0 = 80$

"Closed form" means "explicit"

$Q_1 = 0.08\, Q_0 + 80 = 0.08 \cdot 80 + 80$

$Q_2 = 0.08\, Q_1 + 80 = 0.08^2 \cdot 80 + 0.08 \cdot 80 + 80$

$\vdots$

$Q_n = 0.08^{n-1} \cdot 80 + 0.08^{n-2} \cdot 80 + \cdots + 0.08 \cdot 80 + 80$

$= ax^{n-1} + ax^{n-2} + \cdots + ax + a$, $a = 80$, $x = 0.08$

$= \dfrac{a(1-x^n)}{1-x} = \boxed{\dfrac{80(1-0.08^n)}{1-0.08}}$.

10 (b) $D = 16 + 2\cdot\left[16\cdot\frac{3}{4} + 16\cdot\left(\frac{3}{4}\right)^2 + 16\left(\frac{3}{4}\right)^3\right]$

(c) $D_n = 16 + 2\cdot\left[\underbrace{16\cdot\frac{3}{4} + 16\left(\frac{3}{4}\right)^2 + \dots + 16\left(\frac{3}{4}\right)^{n-1}}\right]$ n-2 terms!

$= a + ax + ax^2 + \dots + ax^{n-2}$

with $a = 16\cdot\frac{3}{4}$

$x = \frac{3}{4}$

So sum $= \dfrac{a(1-x^{n-1})}{1-x}$

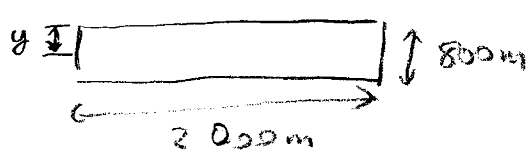
$= \dfrac{16\cdot\frac{3}{4}\left(1-\left(\frac{3}{4}\right)^{n-1}\right)}{1-\frac{3}{4}}$

$\boxed{D_n = 16 + 96\left(1-\left(\frac{3}{4}\right)^{n-1}\right)}$

11 $\displaystyle\sum_{n=0}^{\infty} \frac{8+5^n}{7^n} = \sum_{n=0}^{\infty} \frac{8}{7^n} + \sum_{n=0}^{\infty}\left(\frac{5}{7}\right)^n$

Text

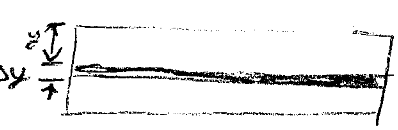
§8.5 #24



(a) 9800 newtons/m^3 · 800m = $\boxed{7{,}840{,}000 \text{ newtons}/m^2}$

(b) At depth y, pressure is $9800\cdot y$.

Force on a slice of thickness Δy



is $9800\cdot y\cdot\Delta y\cdot 2000$. So

Total force $= \int_0^{800} 9800\cdot 2000\cdot y\,dy = \boxed{6.3\times10^{12} \text{ newtons}}$