Math 454 Problem Set #1
Fall 2008 Sect. 1

Due Thursday, 9/4, at the beginning of class.

Reminder. You are encouraged to work together on homework problems. However, the final write-up must be your own. You will be quizzed on some of these problems, so be sure you can do all the problems without help.

Also, unless otherwise stated, all reading assignments are from the text.

Read: Ch. 1; Sects. 2.0-2.4.

Problems (note that the problems for Ch. 2 begin on p. 36 of the text):

- Sect. 2.1 #1, 2
- Sect. 2.2 #2, 4, 8, 9, 10, 13
- Sect. 2.4 #4, 90

P1. Consider the differential equation
\[ \dot{x} = \sin^2\left(\frac{\pi n}{x}\right) + \sin^2(\pi x), \] (1)
where \( n \) is a positive integer. For this problem, we assume that \( x > 0 \). That is, the phase space is the half-line \( \{ x > 0 : x \text{ is a real number}\} \). (This is a natural assumption because the right hand side of Eq. (1) is always \( \geq 0 \), so if \( x(0) > 0 \) then \( x(t) > 0 \) for all \( t \).)

(a) Graph the vector field in Eq. (1), with \( n = 6 \) and \( 1 \leq x \leq 7 \).
(b) What are the fixed points of Eq. (1) with \( n = 6 \)? Classify them in terms of their stability.
(c) For a general positive integer \( n \), what are the fixed points of Eq. (1)?
(d) Let \( n > 2 \) be a given positive integer. Suppose the trajectory of Eq. (1) starting at \( x(0) = 2 \) satisfies \( \lim_{t \to \infty} x(t) = n \). What kind of number is \( n \)?