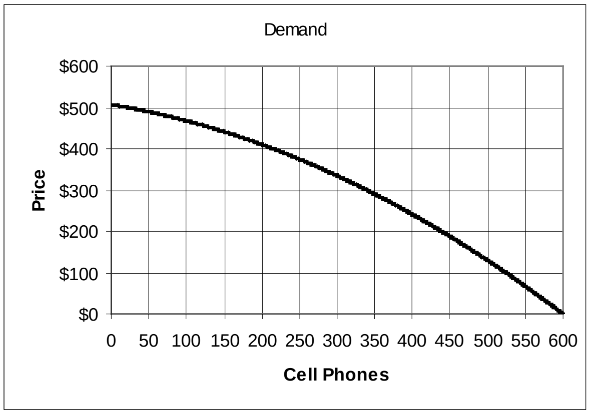
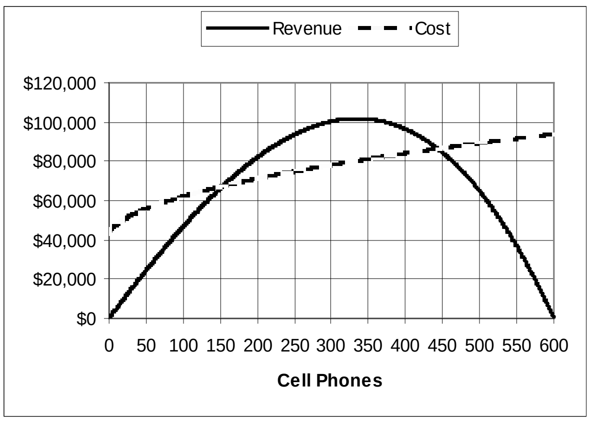
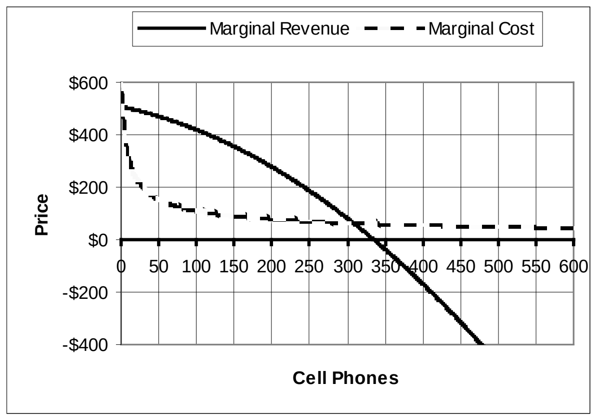
Plot 1



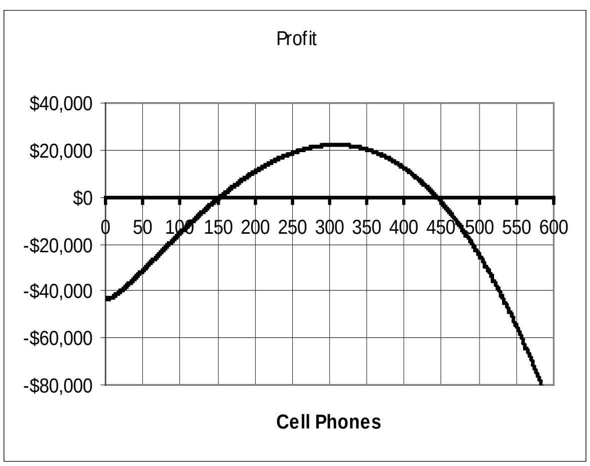
Plot 2



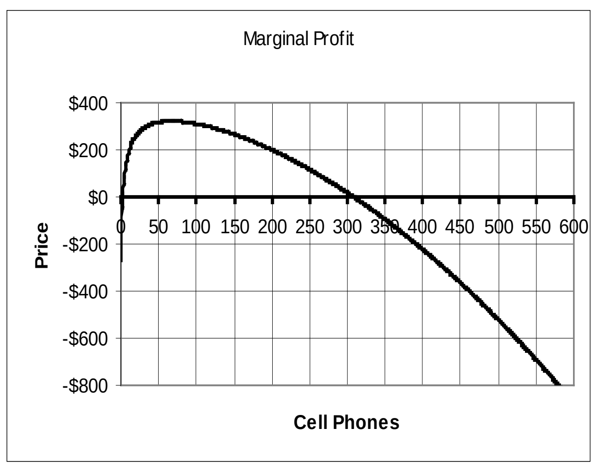
Plot 3



Plot 4



Plot 5



1. Determine which plot(s) (1 through 5) can be used to estimate the following quantities. Then estimate the value of the quantity. Include notation whenever possible.

A. Fixed costs.
$40,000 $C(0) = 40000$
Plot 2: Look for the vertical intercept of the cost function.

B. Quantity that maximizes revenue.
325 cell phones $q \approx 325$
Plot 1: Look for the largest rectangle with sides along the axes and one corner on the graph.
Plot 2: Look for the maximum of the revenue function.
Plot 3: Look for the horizontal intercept of the marginal revenue function.

C. Price for the quantity that maximizes revenue.
$300 $D(325) \approx 300$
Plot 1: Look for value of the demand function when $q \approx 325$.
Plot 2: Look for the revenue when $q \approx 325$, then divide that value by 325.

D. Quantity that makes marginal cost equal marginal revenue
305 cell phones $R'(305) = C'(305)$
Plot 2: Look for the greatest vertical distance between the graphs where $R(q) > C(q)$.
Plot 3: Look for the intersection of the graphs.
Plot 4: Look for the maximum.
Plot 5: Look for the horizontal intercept.

E. Quantity that produces zero profit
150 or 450 cell phones $P(150) = 0$, $P(450)$
Plot 2: Look for the intersections of the graphs.
Plot 4: Look for the horizontal intercepts.

F. Quantity that maximizes profit.
305 cell phones $P'(305) = 0$
Plot 2: Look for the greatest vertical distance between the graphs where $R(q) > C(q)$.
Plot 3: Look for the intersection of the graphs.
Plot 4: Look for the maximum.
Plot 5: Look for the horizontal intercept.

G. Inflection point on the profit graph
60 cell phones
Plot 4: Look for a change in concavity.
Plot 5: Look for a maximum or minimum.

H. Total possible revenue.

$\int_0^{600} D(q)dq$

Plot 1: Look at the total area under $D(q)$ over the interval [0, 600].

I. Consumer surplus when the profit is maximized.

$\int_0^{305} D(q)dq - (305)(330)$

Plot 1: Look at the area of the region between the line $y = 330$ and the graph of $D(q)$ over the interval [0, 305].

J. Other lost revenue

$\int_{305}^{600} D(q)dq$

Plot 1: Look at the area under $D(q)$ over the interval [305, 600].

2. Use $D(q) = -0.0009q^2 - 0.305q + 507$, $VC(q) = 2200\sqrt{q}$, and $C(0) = 41000$ to find equations for each for the following functions.

A. $R(q) = (-0.0009q^2 - 0.305q + 507) \cdot q$

B. $C(q) = 41000 + 2200\sqrt{q}$

C. $P(q) = (-0.0009q^2 - 0.305q + 507) \cdot q - (41000 + 2200\sqrt{q})$

D. $R'(q) = -0.0027q^2 - 0.610q + 507$

E. $C'(q) = 1100q^{-1/2}$

F. $P'(q) = -0.0027q^2 - 0.610q + 507 - 1100q^{-1/2}$

In each case, determine what the symbols or words represent (graphically and practically if possible). In the case where calculations are shown, determine what is being computed.

1. $D(q)\cdot q$

This calculates revenue when q units are sold at a price of $D(q)$ for each unit.

2. $D(50) = \$36.95$

If we want to sell 50 items, we should set the price for an item at \$36.95.

3. The value of q that makes $R(q) - C(q) = 0$

If we have graphs of both $R(q)$ and $C(q)$, this equation can be used to find the intersections between the graphs. If we have a graph of $P(q)$, this equation will correspond to finding the horizontal intercepts of the graph. This would be used to find the quantity of items produced and sold that would generate zero profit (also called a break-even point).

4. The value of q that makes $R'(q) = 0$.

If we have the $R'(q)$ graph, this equation can be used to find the horizontal intercept of the graph. If we have the $R(q)$ graph, this equation corresponds to finding a horizontal tangent. This would be used to find a maximum or minimum revenue. Can also be written as $MR(q) = 0$.

5. The C_0 in the formula $C(q) = 23q + C_0$.

This is the fixed cost. It is a monetary amount that must be paid regardless of how many units are produced.

6. The value of q that makes $R'(q) = C'(q)$

If we have graphs of both $R'(q)$ and $C'(q)$, this equation can be used to find the intersections between the graphs. If we have a graph of $P'(q)$, this corresponds to finding a horizontal intercept. If we have graphs of both $R(q)$ and $C(q)$, this corresponds to finding the quantity where the vertical distance between the two graphs is greatest. If we have a graph of $P(q)$, this corresponds to finding a horizontal tangent and can be used to find a maximum or minimum profit. Can also be written $MR(q) = MC(q)$.

7. $\dfrac{f(a+h) - f(a-h)}{2h}$

This is called a difference quotient. It finds the slope of a line between two points and can be used to estimate $f'(a)$ when h is small.

8. $\lim\limits_{h\to 0}\dfrac{f(a+h) - f(a-h)}{2h}$

This is the definition of $f'(a)$. It is the actual slope of the tangent line to a function $f(x)$ at $x = a$.

9. $\lim_{h \to 0} \frac{f(x+h) - f(x-h)}{h}$

This is the definition of $f'(x)$. It is a function whose *y*-values correspond to slopes on the graph of $f(x)$.

10. $f'(a)$

This is a number. It is the slope of the tangent line to the graph of $f(x)$ at $x = a$.

11. Variable costs.

This is the cost for producing one item. It could be constant or non-constant.

12. Slope of the tangent line to $f(x)$ at $x = a$.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a-h)}{2h}$$

13. Difference quotient.

$$\frac{f(a+h) - f(a-h)}{2h}$$

14. Inflection point.

A point on the graph where the concavity changes (from up to down or down to up).

15. Equation of the tangent line to $f(x)$ at $x = a$.

$y = f'(a)(x-a) + f(a)$ This can be used to estimate values of $f(x)$ near $x = a$.

16. The value of q where the graph of $C'(q)$ has a maximum or minimum.

This corresponds to an inflection point on $C(q)$.

17. Marginal cost

This is the change in cost when an additional unit is produced. $C'(q)$

18. $MP(80) = \$500$ where q is the number of scooters and P is measured in dollars.

We currently produce and sell 80 scooters. What will happen to our profit if we produce and sell 81 scooters instead? Our profit will increase by approximately \$500.

19. Solver

An Excel tool used to approximate maximums, minimums, and intercepts of a function.

20. $f'(x) = n \cdot x^{n-1}$

The derivative rule for power functions.

21. $f'(x) = \ln a \cdot a^{x}$

The derivative rule for exponential functions.

22. $\frac{b-a}{n}$

A formula used to find the distance between consecutive *x* values when plotting *n*+1 evenly spaced values over the interval [*a*, *b*]. Can be used to determine the width of each rectangle when using *n* midpoint rectangles to approximate the area under a curve. Also written as Δx.

23. Consumer surplus

The extra money a company would have received (but didn't) from some buyers because they were willing to pay a higher price for the item than the one that was set.

24. $\int_a^b f(x)dx$

This is a number. It is the area bounded by the graph of $f(x)$ and the *x*-axis over the interval [*a*, *b*]. When $f(x)$ is below the *x*-axis, this number will be negative. Also known as the definite integral.

25. $\sum_{i=1}^{n} D(q)\Delta q$

The sum of the areas of *n* rectangles of height $D(q)$ and width Δq.

26. m_i

A midpoint. Used in determining the height of a midpoint rectangle.

27. $\int_0^b D(q)dq$ where $D(b)=0$.

Total possible (potential) revenue.

28. $\int_0^{100}(-0.3q+500)dq - \int_0^{100} 470dq$

Consumer surplus when 100 items are sold at a price of $470.

29. $S_{40}(f,[5,12])$

The sum of the areas of 40 midpoint rectangles over the interval [5, 12]. Can be used to estimate $\int_5^{12} f(x)dx$.

30. Δx

The width of a rectangle used to estimate the area under a curve. Also the difference between consecutive *x* values when plotting a table of points. $\Delta x = \frac{b-a}{n}$.